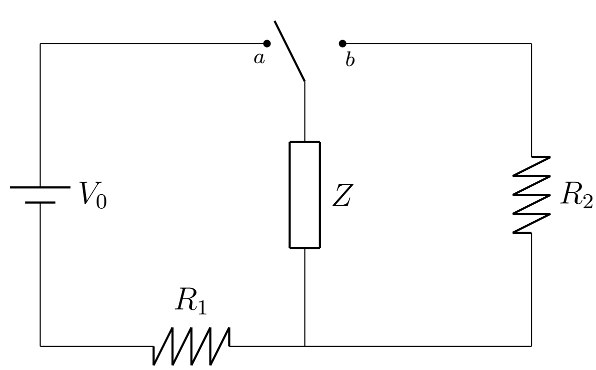
Solutions to Exam 2; Phys 186

1. (70 points) A capacitor stores energy in the electric field between its plates. It takes time for the field to change, so the voltage across a capacitor can't change instantaneously. But the current through a capacitor can change instantaneously, for example when you open or close a switch. All this means that if you have a capacitor with zero electric field, its voltage at that instant has to be zero, but its current can be anything. But when the electric field is at its maximum, the voltage across the capacitor will have the appropriate value, but the current now will have to be zero.

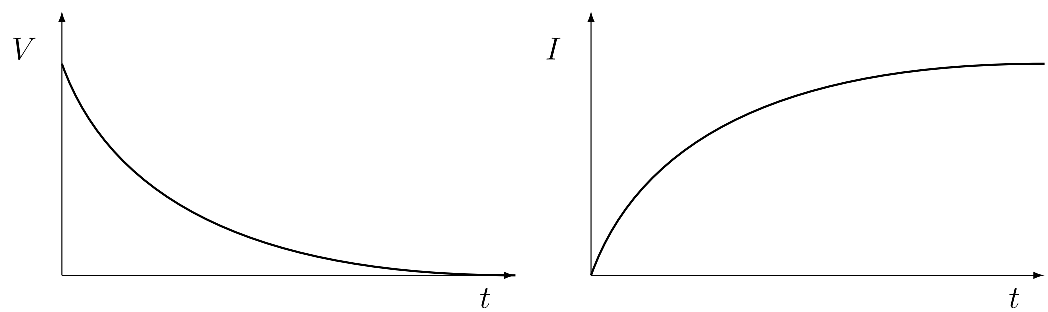
I now give you a device I will call a zinglschritt. A zinglschritt stores energy in the magnetic field in its coils. It takes time for the field to change, so the current through a zinglschritt can't change instantaneously. But the voltage across a zinglschritt can change instantaneously, for example when you open or close a switch. All this means that if you have a zinglschritt with zero magnetic field, its current at that instant has to be zero, but its voltage can be anything. But when the magnetic field is at its maximum, the current through the zinglschritt will have the appropriate value, but the voltage now will have to be zero.

You then have the following circuit that builds up or brings down the magnetic field in a zinglschritt, depending on whether the switch is at position a or b. The rectangle represents your zinglschritt.



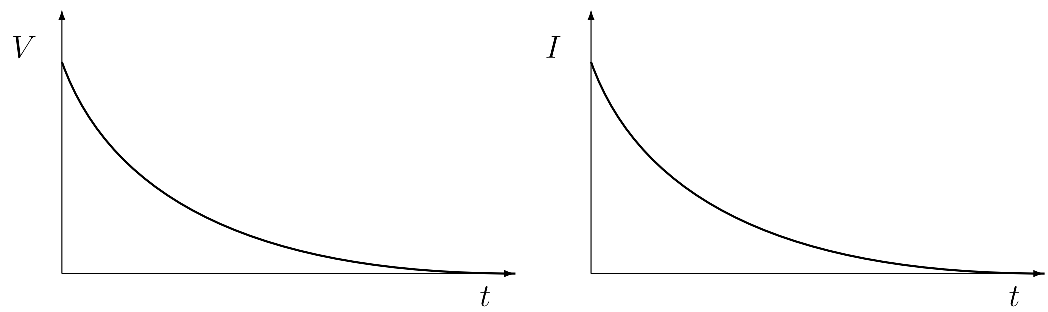
(a) You start with no magnetic field in the zinglschritt, with the switch neither connected to a nor b. Then you connect it to a. Immediately after the switch is set to a, what is the current through the zinglschritt, and the voltage across the zinglschritt? You then wait a long time, so that the magnetic field reaches its maximum value. What, then, is the current through the zinglschritt, and the voltage across the zinglschritt? Make rough sketches of how the voltage and current depend on time, with $t = 0$ as the time you set the switch to a. Explain your reasoning, or provide calculations.

Answer: The loop equation when the switch is at a is $V_0 = V_z + R_1 I$. At $t = 0$, the current $I = 0$. This means that $V_z = V_0$ at the beginning. After a long time passes, $V_z = 0$, so the current becomes $I = V_0/R_1$. The curves should be such that the voltage and current approach their long time values gradually, not abruptly.

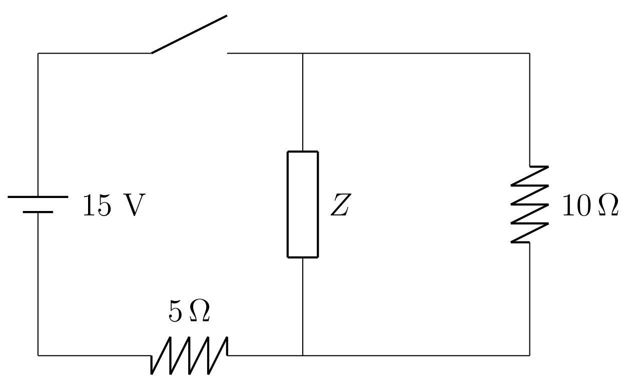


(b) After having built up the magnetic field in the zinglschritt with the switch at a for a long time, you then flip the switch to b. Immediately after the switch is set to b, what is the current through the zinglschritt, and the voltage across the zinglschritt? You then wait a long time. What, then, is the current through the zinglschritt, and the voltage across the zinglschritt? Make rough sketches of how the voltage and current depend on time, with $t = 0$ as the time you set the switch to b. Explain your reasoning, or provide calculations.

Answer: The loop equation with the switch is at b is $V_z = R_2 I$. When $t = 0$, the current $I = V_0/R_1$, since the current through a zinglshritt cannot change rapidly. This means that $V_z = R_2 V_0/R_1$ at the beginning. After a long time passes, $V_z = 0$, so the current also becomes $I = 0$.



(c) You now have the following circuit, which includes a zinglschritt. The switch has been open for a long time. You then close the switch at time $t = 0$. Calculate the voltage across the $10\,\Omega$ resistance at time $t = 0$, when the switch has just closed, and then the voltage after a long time has passed and the magnetic field in the zinglschritt has reached its maximum value. Then sketch a graph of how the voltage across the $10\,\Omega$ resistance depends on time.

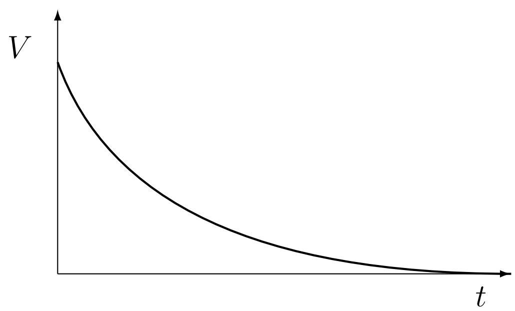


Answer: The junction equation: $I_1 = I_2 + I_3$, where I_1 is the current through the battery and the $5\,\Omega$ resistance, I_2 is the current through the zinglschritt, and I_3 is the current through the $10\,\Omega$ resistance.

The loop equations with $V = RI$ for the resistors: $15\,\mathrm{V} = V_z + (5\,\Omega)I_1$, and $V_z = (10\,\Omega)I_3$.

At $t = 0$, we have $I_2 = 0$. Therefore, $I_1 = I_3$. Putting this into the loop equations, we get $15\,\mathrm{V} = (10\,\Omega)I_1 + (5\,\Omega)I_1$, giving $I_1 = 1\,\mathrm{A}$. This means that the voltage across our device is $(10\,\Omega)(1\,\mathrm{A}) = 10\,\mathrm{V}$.

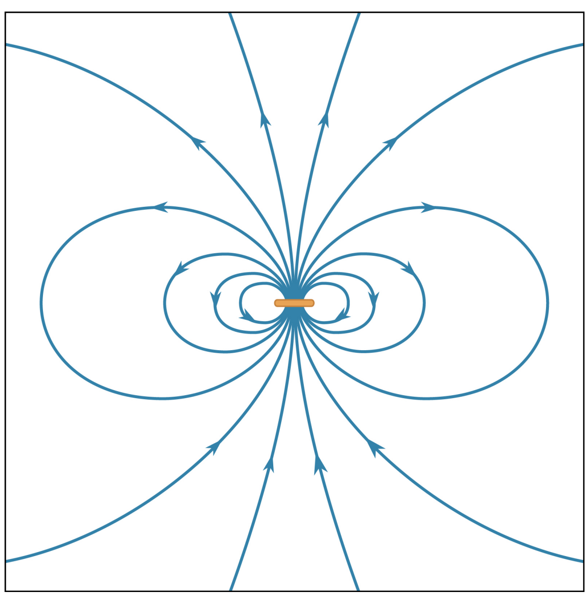
At long times, we have $V_z = 0$. Therefore the device voltage is also 0.



2. (30 points) You have a circular ring which has a constant current I circulating around. This creates a magnetic field.

(a) Draw the magnetic field lines for such a ring from a side view. The ring is perpendicular to the page, and the picture shows a section of the ring through its middle. The current direction is indicated by the cross and dot.

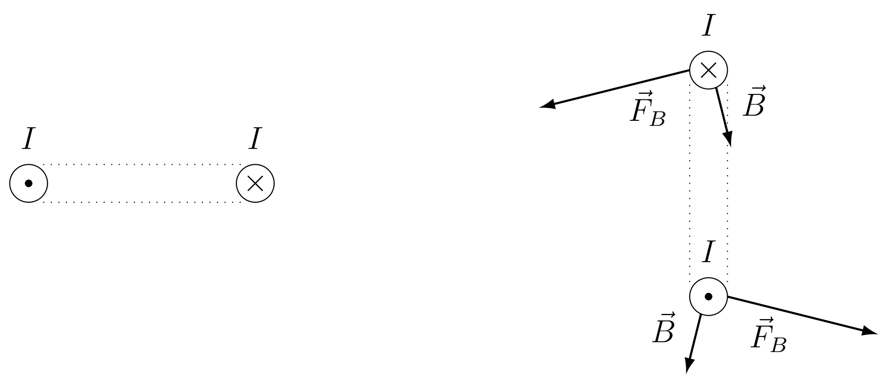
Answer: You can use my drawing from when we did this as an example in class, or look it up online. For a change, let's say you looked it up online; you'd get something like:



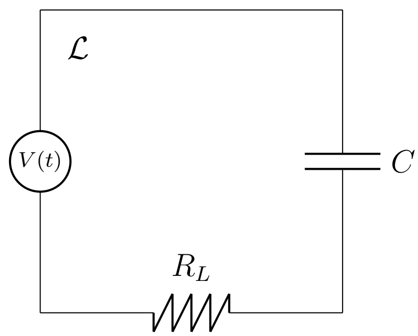
You need to be careful about the direction of the arrows; use the right hand rule.

(b) You now put a second, identical current loop further to the right, with a perpendicular orientation. On each end of the second loop (the cross end and the dot end), draw arrows indicating (i) the magnetic field from the first loop, (ii) the magnetic force that end of the loop will feel. If the second loop is free to move, what do you think will happen to it?

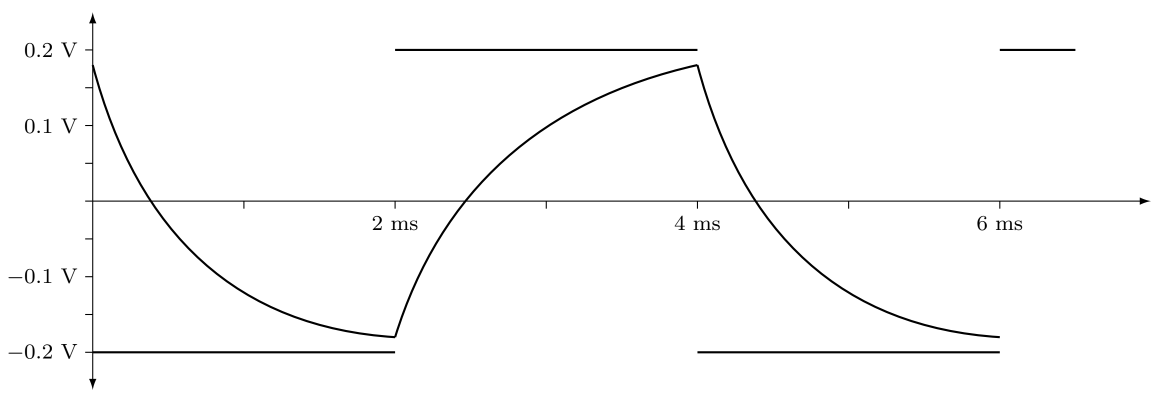
Answer: Using the field lines that we already have, we can figure out the direction of the magnetic field, and then using the right hand rule, the direction of the forces. The second loop will be dragged down slightly, but the main effect will be to rotate it.



3. (50 points) You have two circuits next to each other:

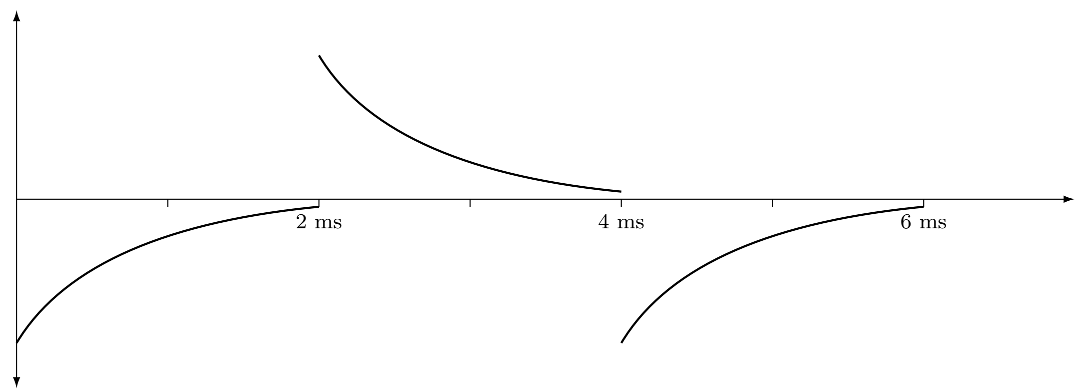


Circuit $\mathcal{L}$ has a voltage source which is a function generator that produces a square waveform $V(t)$, which looks like the following on an oscilloscope:



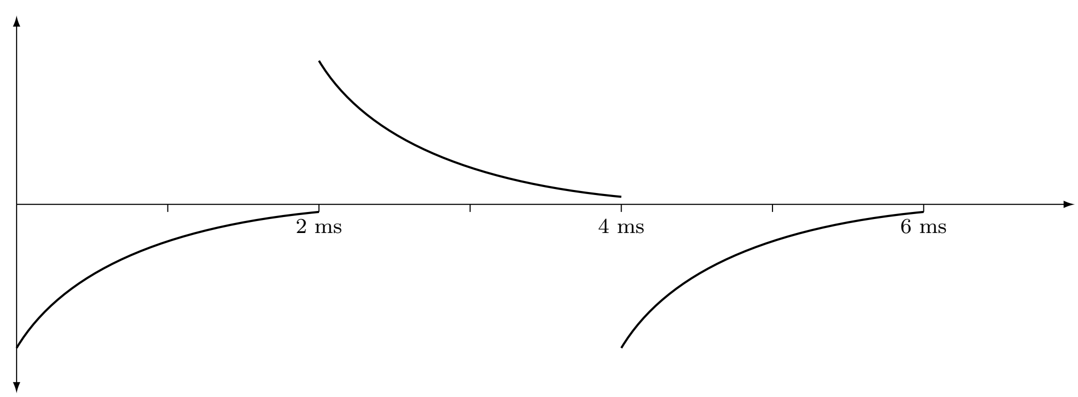
(a) You have the frequency of the function generator set such that the amplitude of the voltage across the capacitor is about 0.18 V (90% of the amplitude of the source voltage). Sketch the shape of the waveform you will see if you measure *the current* in circuit $\mathcal{L}$. Don't put in any numbers—just sketch the waveform. Explain how you arrived at your conclusion.

Answer: I've drawn the capacitor voltage waveform V_c on the upper graph as well. The loop equation for circuit $\mathcal{L}$ is $V(t) = V_c + R_1 I_L$, which means, since the resistance is constant, that $I_L \propto (V(t) - V_c)$. You have to draw the shape of the *difference* between the two voltage waveforms.



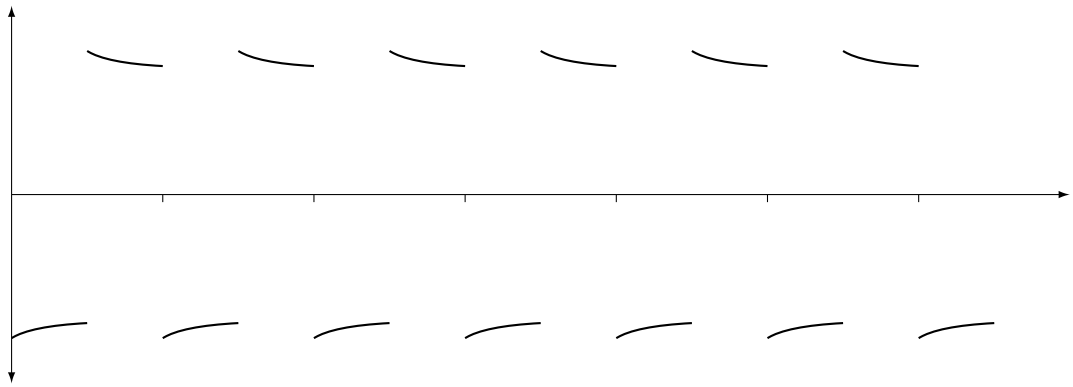
(b) Now sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{R}$ under these conditions. Explain how you arrived at your conclusion.

Answer: I_R is a current induced in circuit $\mathcal{R}$ due to the changing magnetic flux. The magnetic field is produced by I_L. The only thing that is changing is I_L; anything else is a constant that does not affect the shape of the curve. Therefore $I_R \propto -\frac{d}{dt}I_L$. Therefore the shape of this current can be read off the changing slope of the I_L graph above. The graph ends up looking very similar.



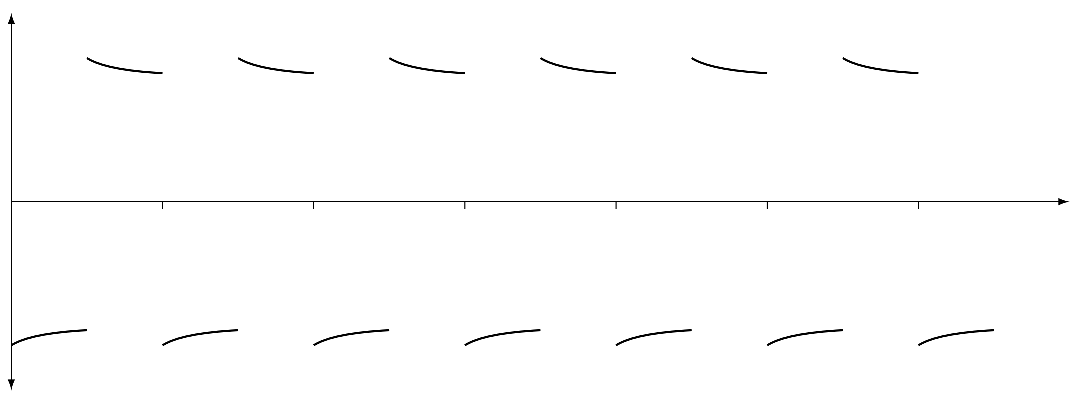
(c) You have the frequency of the function generator set such that the amplitude of the voltage across the capacitor is about 0.02 V (10% of the amplitude of the source voltage). Sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{L}$.

Answer: The reasoning here is very similar. Now the current shape is still the voltage difference, but the frequency is higher and the amplitude of V_c is much smaller.



(d) Now sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{R}$ under these conditions.

Answer: Again, very similar. You may remember from calculus that the derivative of an exponential is an exponential. Capacitors charging up and discharging are described by exponentials.



4. (50 points) Say you're doing the lab where you accelerated and shot a beam of electrons onto a screen. The mass of an electron is $m_e = 511\,\text{keV}/c^2$.

(a) You accelerated the electrons through a voltage difference of up to 5.00 kV on your dial. At $V_a = 5.00\,\text{kV}$, then, what is the kinetic energy of the electrons in the beam, in units of keV? *Hint:* 1 eV is literally the electron charge magnitude e multiplied by 1 V. Therefore, you shouldn't need any real calculation to get this answer.

Answer: $e(5\,\text{kV}) = 5\,\text{keV}$.

(b) What fraction of the speed of light are these electrons traveling? Use $\frac{1}{2}m_e v^2$ for your kinetic energy, as in your homework and the lab.

Answer: The usual energy conservation gives

$$5\,\text{keV} = \frac{1}{2}(511\,\text{keV})\frac{v^2}{c^2} \quad \Rightarrow \quad \frac{v}{c} = \sqrt{\frac{2 \cdot 5}{511}} = 0.140$$

14% of the speed of light. I mentioned that our electrons in the lab were fast.

(c) A more accurate calculation of the speed would go like this. The total energy of the electron is $\gamma m_e c^2$, while the energy of an electron at rest, where $\gamma = 1$, is $m_e c^2$. Therefore, the kinetic energy, which is the additional energy an object has due its motion, must be the difference: $K = (\gamma - 1)m_e c^2$. Use this relativistic form of kinetic energy to recalculate the fraction of the speed of light the electron has.

Answer: Here, γ depends on v/c, so after some algebra,

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{K}{m_e c^2}\right)^2}} = 0.139$$

Again, 14% of the speed of light.

(d) Compare your results in (b) and (c). Do you think relativity was important enough to account for in your lab?

Answer: The answers are very similar; you did not need relativity. While very fast, the electrons are not close enough to the speed of light for relativistic effects to become important.

(e) Say you got a lot more expensive equipment that could provide an accelerating voltage of up to $V_a = 500\,\text{kV}$. In that case, what would you calculate the speed of the electrons to be (as a fraction of the speed of light) if you used $K = \frac{1}{2}m_e v^2$?

Answer: Same calculation:

$$500\,\text{keV} = \frac{1}{2}(511\,\text{keV})\frac{v^2}{c^2} \quad \Rightarrow \quad \frac{v}{c} = \sqrt{\frac{2 \cdot 500}{511}} = 1.40$$

140% of the speed of light. That should raise an eyebrow.

(f) Redo the calculation for v as a fraction of the speed of light with $V_a = 500\,\text{kV}$, but now using $K = (\gamma - 1)m_e c^2$.

Answer: Same calculation:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{K}{m_e c^2}\right)^2}} = 0.863$$

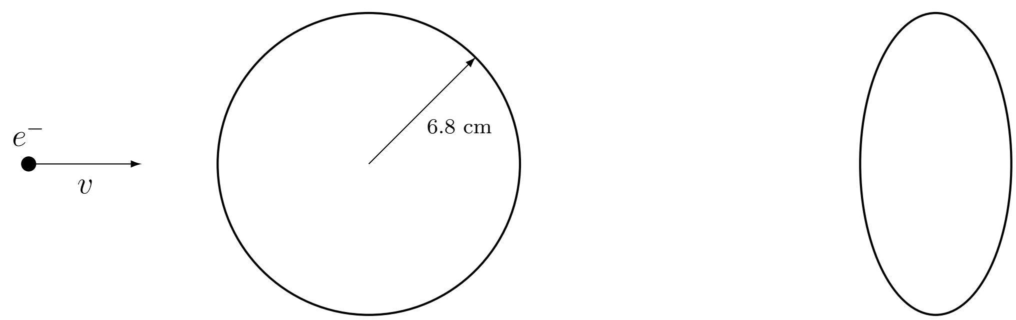
86% of the speed of light.

(g) Compare your results with different expressions for kinetic energy in (e) and (f) and interpret what they mean.

Answer: Clearly the nonrelativistic calculation is wrong: faster than light? The electrons are now moving close enough to the speed of light that relativity becomes important, and the proper relativistic kinetic energy expression is necessary.

(h) Again, $V_a = 500\,\mathrm{kV}$. In the lab reference frame, the copper coils with the current providing the magnetic field were circles with a radius of about $R = 6.8\,\mathrm{cm}$. Sketch how the coils look in the electrons' reference frame, and calculate the appropriate dimensions (height, width) for the coil in that frame.

Answer: Here, $\gamma = 1.98$. The coil is not moving in the lab frame, therefore its dimensions are proper lengths. Length contraction will occur along the direction of motion, so the width will contract down to $2 \cdot 6.8/1.98 = 6.9\,\mathrm{cm}$. The height is perpendicular to the direction of motion, so this will not be contracted, remaining at $2 \cdot 6.8 = 13.6\,\mathrm{cm}$.



lab frame electron frame