Solutions to Assignment 3; Phys 186

1. (30 points) You pass white light through a prism, and see the ray of light split into a rainbow pattern. Choose between four possible explanations:

(a) The prism is a diffraction grating, with the spaces between atoms acting as slits.

(b) The prism is a double slit, with the spaces between atoms acting as slits.

(c) The prism has an index of refraction n that depends on the wavelength λ, with n increasing as λ increases.

(d) The prism has an index of refraction n that depends on the wavelength λ, with n decreasing as λ increases.

Explain how each of the following is relevant to your decision:

- Atoms in a crystal such as the prism are separated by distances around 1 nm.

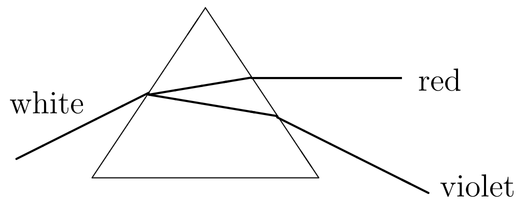
  Answer: Since visible light has a wavelength about 500 nm, a diffraction grating or double slit with a slit separation of 1 nm won't work very well to give a rainbow pattern. After all, $\sin\theta = m\,\lambda/d$, and $\lambda/d \approx 500$—the $m = \pm 1$ peak does not exist. Another way of putting it is that $d \ll \lambda$, so you won't see much of wave-like effects such as interference to produce the rainbow.

- No white light gets through; only the rainbow pattern is seen.

  Answer: If the prism was acting as a double slit or a diffraction grating, you'd also see a strong central maximum of white light. You don't.

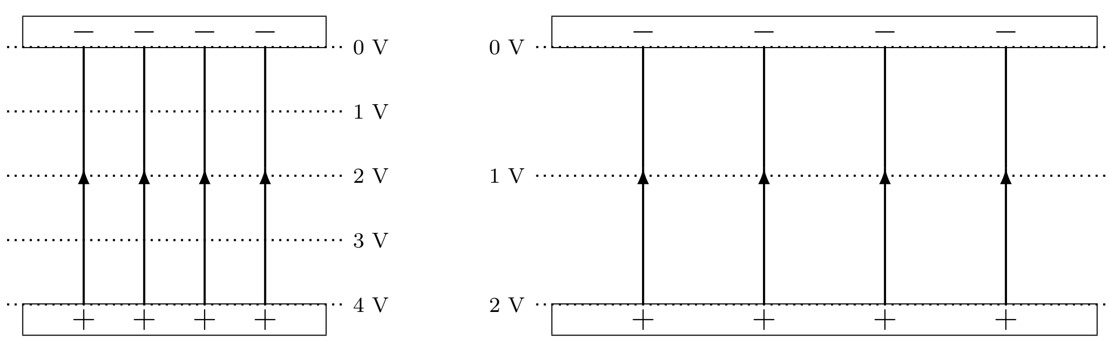
- Red light appears at the top, violet at the bottom:

  Answer: You can see that red light ray (long wavelength λ) is bent less than the violet (short wavelength λ). The larger the index of refraction n, the stronger the refraction you will observe. Therefore n must depend on λ and n must *decrease* with increasing λ.



2. (30 points) Use the rules concerning equipotential lines and electric field lines to figure out what happens when you double the plate area of a parallel plate capacitor. You have a capacitor with plate area A and plate separation d with charges $\pm Q$ on its plates, with a voltage difference of 4.0 V between the plates. You also have a capacitor that is identical in every respect, except that the same charge is distributed throughout double the area, $2A$. What will the voltage reading on the second capacitor be? Draw electric field lines and equipotential lines at 1 V intervals, and explain your reasoning.

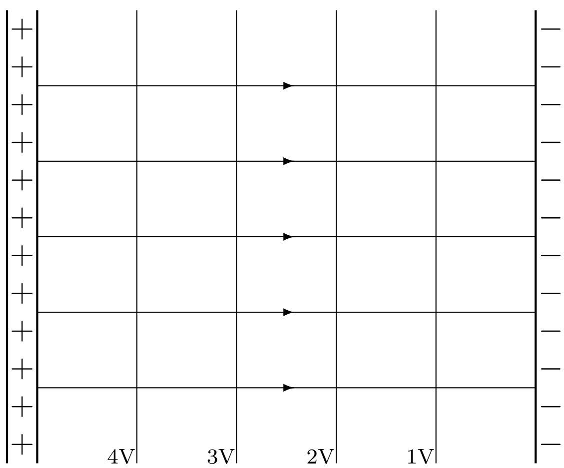
Remember that the the magnitude of the uniform electric field produced by an infinite plane of charge is proportional to the charge density (charge per unit area).



Answer: If the area is doubled while the amount of charge remains the same, the density of electric field lines will be halved. Since the electric field strength is proportional to the line density, this means the electric field strength will be halved. You can indicate this by drawing the electric field lines further apart. But if the lines are twice as far apart, that means the equipotential lines are spaced father apart, to give a more gentle slope of descent. In fact, they have to spaced twice as far apart, so the voltage can only go from 0 V to 2 V over the plate separation distance.

3. (40 points)

(a) You have a parallel plate capacitor. The left plate is set at a potential of 5 V, the right at 0 V. Draw in equipotential lines in between at 1, 2, 3, and 4 V, and the electric field lines. Finally, find an expression for the *total electrical force* by which the plates are attracted toward one another, in terms of the constant ϵ_0, the charge Q on each plate, the plate area A, and the plate separation d.



Answer: Each charge q on the plate will be in the constant electric field due to the other plate, with magnitude $E = Q/(2\epsilon_0 A)$—half of the full field. Therefore it will feel a force of magnitude $F = qQ/(2\epsilon_0 A)$. The total force is, since each charge feels the same force,

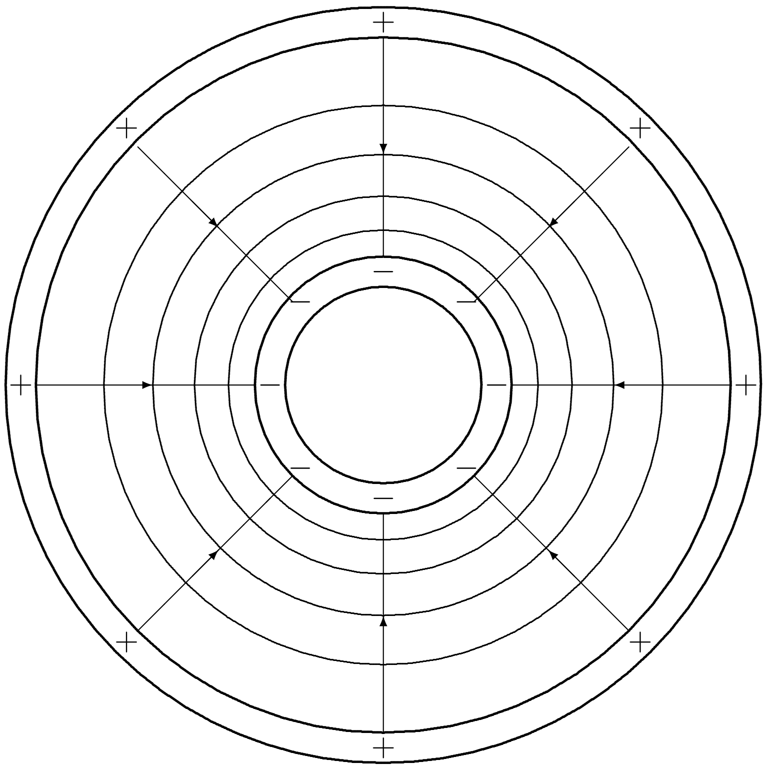
$$\sum F = \left(\sum q\right) \frac{Q}{2\epsilon_0 A}$$

But the total charge on a plate is $\sum q = Q$. Therefore

$$\sum F = \frac{Q^2}{2\epsilon_0 A}$$

and is attractive. (The plates are attracted to one another.)

(b) You now have a cylindrical capacitor: two concentric metal rings with equal and opposite charges. Draw a qualitative map of the equipotential lines and electric field lines for this case. Also include the voltage and electric fields inside the inner ring. What is the total electrical force acting on each ring?



Answer: From the symmetry, the electric field lines will be radial in direction, from the positively charged outer ring toward the negatively charged inner ring. Since the electric field lines will not be parallel, but will be getting closer together as they approach the inner ring, that means the electric field magnitude must be getting larger as you approach the inner ring. From all this, we can also figure out the equipotential lines. They are perpendicular to the electric field lines, which means they have to be concentric circles. And since the electric field magnitude becomes larger when the equipotential lines are close to one another, these circles must be incleasingly close to one another as you get closer to the inner ring.

The voltage is constant inside the inner ring, and the electric field is zero. The total force is zero as well, from the symmetry of the situation: any charge on the inner ring that is attracted to the outer ring has a counterpart on the other side of the circle that feels an equal force in the opposite direction.