Solutions to Exam 1; Phys 186

1. (20 points) We have been discussing an ideal spring, where there is no friction or drag to dissipate energy. You have the following six proposals for a more realistic model. The parameter $\tau > 0$ in all cases. Identify the best model, and explain why it works and the others are wrong. *Hint:* It might help to sketch the proposed functions.

(a) $y(t) = A\, e^{t/\tau} \cos(\omega t + \phi)$

Answer: This includes an amplitude $Ae^{t/\tau}$ that *grows* in time, which, since the spring total energy is $\frac{1}{2}kA^2$, means continual addition of energy to the spring system. So this cannot be correct.

(b) $y(t) = A\, e^{-t/\tau} \cos(\omega t - \phi)$

Answer: Here, the amplitude $Ae^{-t/\tau}$ decreases, indicating continual loss of energy. Exactly what we want. The minus sign in the phase $-\phi$ is irrelevant: all it does is shift the graph along the t-axis, to accommodate differing starting conditions.

(c) $y(t) = A \cos(\tau t^2 + \omega t + \phi)$

Answer: Adding the τt^2 term to the cosine means that the frequency of the oscillations is no longer constant. While interesting, this is not something that is easy to achieve with a spring, and it does not reflect a dissipation of energy.

(d) $y(t) = A \cos(-\tau t^2 + \omega t + \phi)$

Answer: Much the same here, the negative $-\tau t^2$ does not alter the argument.

(e) $y(t) = A^2 \cos(\omega t + \phi^2)$

Answer: We're just calling the amplitude A^2 rather than A here. Other than the strange labeling, this is exactly the same as the behavior of an ideal spring.

(f) $y(t) = A^{1/2}\cos(\omega t + \phi^{1/2})$

Answer: We're just calling the amplitude $A^{1/2}$ rather than A here. Other than the strange labeling, this is exactly the same as the behavior of an ideal spring.

2. (30 points) Your microwave optics lab included a double-slit experiment where $d = 5.5\,\text{cm}$ and $\lambda = 3.0\,\text{cm}$. You were only able to observe the $m = 0, \pm 1$ peaks of intensity.

(a) With a different d, you might have been able to observe the $m = \pm 2$ peaks. Find a d value that would allow you to see these peaks but not $m = \pm 3$. Make reasonable assumptions about what you need to be able to see peaks with the setup we used.

Answer: To be able to see the $m = \pm 2$ peaks, $\sin\theta_2 = 2\lambda/d$ must have a solution for θ_2. That means $2\lambda/d = 1$, for an angle of $\theta_2 = 90°$. This requires a minimum of $d = 2\lambda = 6.0\,\text{cm}$.

We don't want to see $m = \pm 3$, and for $\sin\theta_3 = 3\lambda/d$ not to have a solution, the maximum possible $d = 3\lambda = 9.0\,\text{cm}$.

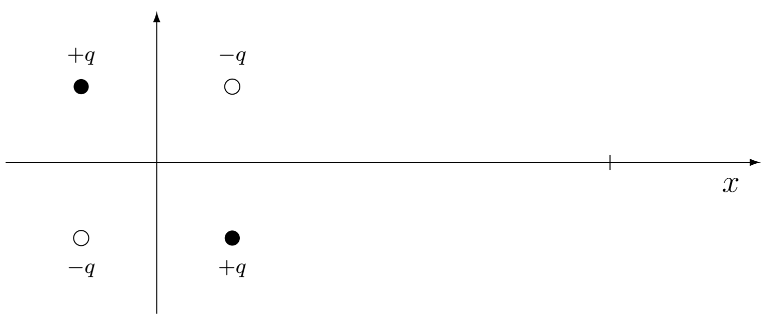
Now, a peak just at 90° would not really be visible in the lab. So pick a d value somewhere in between: say $d = (9 + 6)/2 = 7.5\,\text{cm}$. Then, the angle for $m = 2$ will be $\theta_2 = \sin^{-1}(6/7.5) = 53°$, which should be easily observable.

(b) If you had a diffraction grating instead of two slits, the intensity peaks would have been sharper, making a more precise experiment. Why do you think I didn't give you a grating? (For example, do you think its cost might have been too high?)

Answer: A proper grating requires lots—thousands at least—of slits. With a slit spacing around 5.5 cm, the diffraction grating required would be over 50 m long—hardly the kind of thing to fit on a tabletop in our lab.

3. (30 points) Four charges are arranged on a square with sides a, and the origin of the axes is at the center of the square.

(a) Find the x- and y-components for the total electric field at an arbitrary point on the x-axis, where $x > a/2$. Also draw an arrow depicting the total electric field vector at this point. *Hint:* You can do a brute-force calculation for four charges. Or you can do next to no calculation if you've kept good notes.



Answer: This is a combination of two dipoles, exactly like what we solved in class, except that one of them is upside down and they are displaced by $\pm a/2$ along the x-axis. Therefore, we use the single dipole result, $E_x = 0$ and $E_y = -kqa[x^2 + (a/2)^2]^{-3/2}$.

For the dipole on the right, we take the negative of the results, since the + and − charges are reversed, and replace x with $x - a/2$. For the dipole on the left, we replace x with $x + a/2$, and add the components. $E_x = 0$ and

$$E_y = kqa\left[\frac{1}{\left((x-\frac{a}{2})^2+(\frac{a}{2})^2\right)^{3/2}} - \frac{1}{\left((x+\frac{a}{2})^2+(\frac{a}{2})^2\right)^{3/2}}\right]$$

There's no need to simplify.

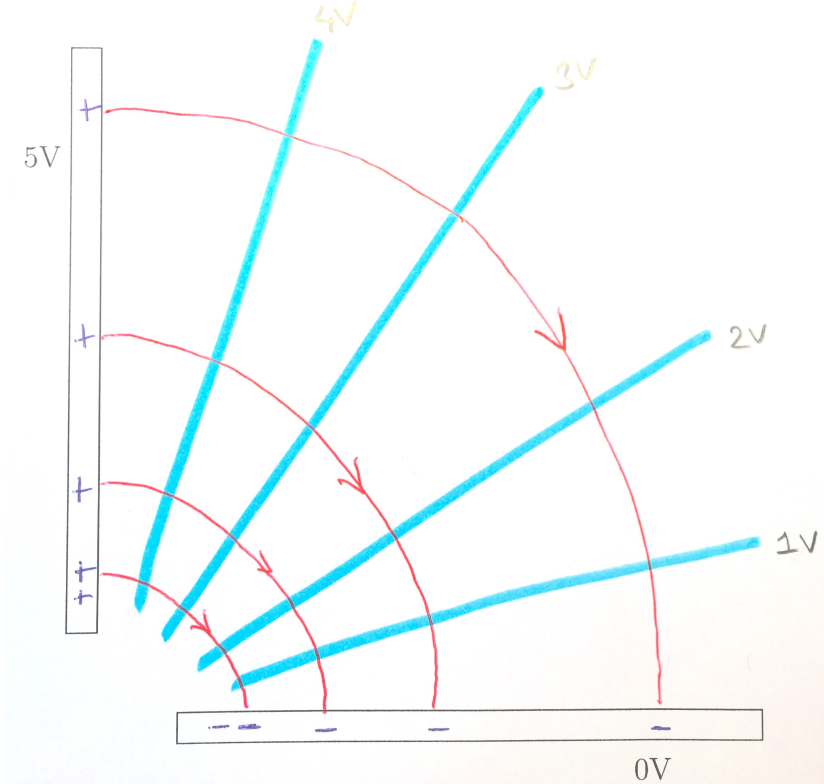
(b) When $x \gg a$, the electric field magnitude behaves like $E \propto 1/x^n$. What is n? *Math hint:* $[(x+\alpha)^2 + \beta^2]^{-3/2} \approx x^{-3}(1 - 3\alpha/x)$ when $|\alpha| \ll x$ and $|\beta| \ll x$.

Answer: We can use the math hint with the expression above, with $\alpha = -a/2$ and then $\alpha = a/2$:

$$E_y \approx \frac{kqa}{x^3}\left[\left(1 + \frac{3a}{2x}\right) - \left(1 - \frac{3a}{2x}\right)\right] = \frac{3kqa^2}{x^4}$$

Therefore $n = 4$. Note that for a bare charge, $n = 2$, and for a dipole, $n = 3$. This electric field is even weaker than a dipole; it's known as a *quadrupole*.

4. (20 points) In your equipotential lines lab, say you set up two of the metal plates as shown. You set the voltage of the plate drawn vertically here to 5.0 V, and the voltage of the horizontal one to 0.0 V.



(a) On the diagram above, draw what you think the equipotential lines will look like, with lines for 1V, 2V, 3V, 4V. You need not be concerned about any place other than the space between the plates.

(b) Now add the electric field lines.

(c) There will be an excess of + charges on the 5V plate, and an excess of − charges on the 0V plate. Draw in some + and − charges on each of these plates in a way that you think would produce the field lines that you drew.

(d) If you increase the voltage difference to a large value, you may see sparks jumping between the plates. Where would the sparks jump across? Explain why.

Answer: The + and − charges will be attracted to one another. As the voltage increases, so does the electric field strength, and at some point, the attractive force will be large enough to rip the charges (electrons in this case) out of the metal and cross the gap. The electric field, and hence the electric force, is strongest where the electric field lines (or, alternatively, the equipotential lines) are closest to one another. That is where the plates are closest to one another.