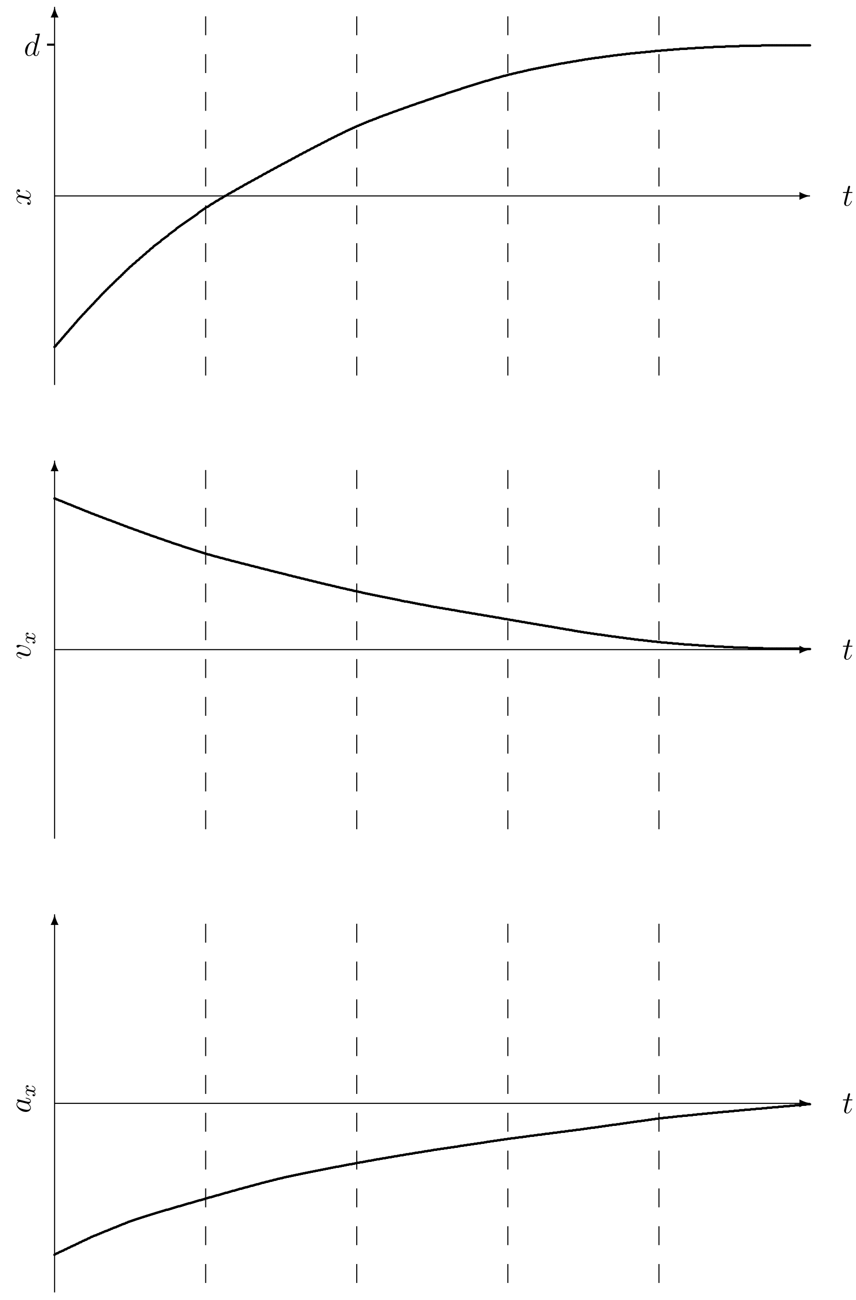
Solutions to Assignment 1; PHYS 185

1. (10 points) The top graph displays how position depends on time for an object that gradually, but ever more slowly, approaches $x = d$. Make a qualitative sketch of the corresponding velocity versus time and acceleration versus time graphs for this motion.



Answer: Notice that the slope of the x-t graph is positive, but that it becomes flatter—the slope gets closer to zero—ass t increases. Therefore the velocity graph stays positive, but decreases.

We look at slopes again to find the acceleration. The velocity is *decreasing*, therefore the acceleration is negative. But since v_x flattens out with time, a_x also approaches zero.

2. (30 points) The distance from London to Sydney is 1.70×10^7 m. You take a supersonic flight that covers this distance in exactly $T = 2.00$ hours. Say the flight takes a time t_a to accelerate from rest, reaching a constant cruising speed of v_c, and then while landing, takes the same time t_a to decelerate. For a commercial flight that will be taken by people with varying health conditions, the magnitude of the horizontal component of the acceleration imposed on the passengers should not exceed $2g$ ($g = 9.80$ m/s^2) for longer than three minutes. Calculate t_a for this maximum $a_x = 2g$, and determine whether this flight can be safe.

Answer: The distance covered, let's call it d, is the area under the v-t curve. (You can also do this by treating the three different constant acceleration segments of the motion separately, using the standard motion with constant a_x equations, and adding all of it up, but it amounts to the same thing.)

$$\Delta x = v_c(T - t_a)$$

Also, from the graph,

$$a_x = \frac{v_c}{t_a}$$

Setting $a_x = 2g$ and combining the two equations, we get a quadratic equation for t_a:

$$d = 2gt_a(T - t_a) \quad \Rightarrow \quad t_a^2 - Tt_a + d/(2g) = 0$$

The solutions to this are, using $d = 1.70 \times 10^7$ m and $T = 7200$ s,

$$t_{a\,1,2} = \frac{T \pm \sqrt{T^2 - 2d/g}}{2} = 123 \text{ s}, 7077 \text{ s}$$

The first answer, 123 s, is what we need (the other is $T - t_a$). This is a reasonable value, under three minutes, so this should be safe.

3. (30 points) You have a cannon that launches rubber balls with an initial speed of $v_0 = 12.6$ m/s. You set it at an angle $\theta = 38°$ above the horizontal, and shoot a ball at a high vertical wall standing a distance $l = 9.20$ m in front of the cannon.

(a) Find symbolic expressions for v_x and v_y at the instant before the rubber ball hits the wall. Then plug in the numbers and find their values.

Answer: The initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. The acceleration components are $a_x = 0$ and $a_y = -g$. We want to find v_x and v_y at the time t when $x = l$. Therefore,

$$l = 0 + v_0 \cos\theta\, t + 0 \qquad \Rightarrow \qquad t = \frac{l}{v_0 \cos\theta}$$

At this time,

$$v_x = v_{0x} = v_0 \cos\theta = 9.93 \text{ m/s}$$

and

$$v_y = v_0 \sin\theta - gt = v_0 \sin\theta - \frac{gl}{v_0 \cos\theta} = -1.32 \text{ m/s}$$

(b) The instant *after* the rubber ball bounces off the wall, the y-component of its velocity remains the same as it was just before it hit the wall. But the x-component of its velocity reverses its direction (same magnitude, opposite sign). Find out where, relative to the cannon, the ball falls back to the ground.

Answer: There are multiple ways to solve this. The easiest is to recognize that reversing v_x means that the motion after the ball hits the wall will be the same as if the wall were not there, but in the $-x$ direction instead. The distance traveled without the wall would be

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

so the ball would have traveled an extra distance of $R - d$. Since the direction is reversed, we *subtract* that from d, find in that ball landed a distance

$$d - (R - d) = 2d - R = 2d - \frac{2v_0^2 \sin\theta \cos\theta}{g} = 2.7 \text{ m}$$

in front of the cannon.

4. (30 points) You launch a projectile on a level surface on a planet with acceleration due to gravity g, starting from $x_0 = y_0 = 0$, with initial speed v_0 and angle θ with the x-axis. But you're facing a strong horizontal wind, so that the motion has a non-zero $a_x = -w$, where w is a positive constant that stands for the magnitude of the acceleration due to the wind.

(a) Write down the equations for motion along the x and y-axes:

$$v_x(t) = v_0 \cos\theta - wt \qquad x(t) = v_0 \cos\theta\, t - \tfrac{1}{2}wt^2$$
$$v_y(t) = v_0 \sin\theta - gt \qquad y(t) = v_0 \sin\theta\, t - \tfrac{1}{2}gt^2$$

(b) Find the *range* of the projectile: an equation for how far it will travel until it hits the ground again.

Answer: You want t for $y = 0$. The wind has no effect on this; you end up with the usual

$$0 = v_0 \sin\theta\, t - \tfrac{1}{2}gt^2 \qquad \Rightarrow \qquad t = 0 \text{ or } \frac{2v_0 \sin\theta}{g}$$

where you throw away the $t = 0$ solution. The distance traveled is x at this t:

$$x = v_0 \cos\theta \frac{2v_0 \sin\theta}{g} - \tfrac{1}{2}w\left(\frac{2v_0 \sin\theta}{g}\right)^2 = \frac{2v_0^2 \sin\theta}{g}\left(\cos\theta - \frac{w}{g}\sin\theta\right)$$

(c) Check your result: when you set $w = 0$, you should get the same equation for the range as you have in your class notes.

Answer: With $w = 0$, you get

$$x = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

which is what you should have.

(d) The range is positive when $w <$ [an expression involving g and θ]. Find this inequality. Would it make physical sense for the range to be negative?

Answer: Looking at the equation, you see that $x > 0$ when

$$\left(\cos\theta - \frac{w}{g}\sin\theta\right) > 0 \qquad \Rightarrow \qquad w < g\cot\theta$$

The wind can be strong enough that the projectile loops backward.

(e) See what happens when $w = g$ and $\theta = 45°$. Interpret your result in this case—what does the motion look like?

Answer: In this case the range ends up as zero. The total acceleration vector is toward the origin at at a 45° angle—you shoot your projectile straight into that; it goes diagonally up a bit and comes straight back down.