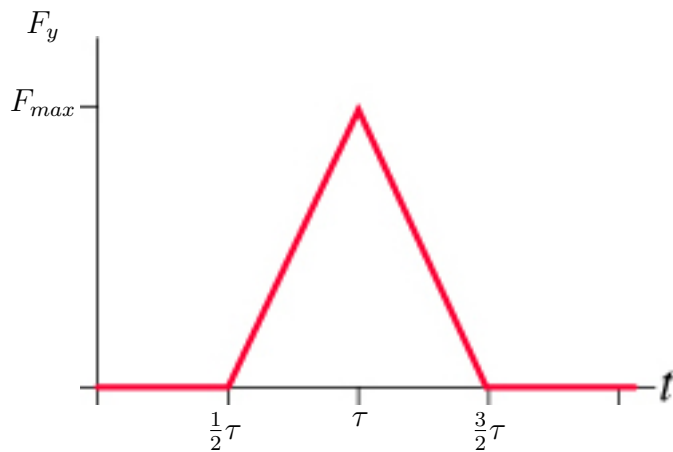


1. (30 points) A ball with mass m , starting at rest, is dropped from a height of h_i and bounces on a hard floor. The force on the ball from the floor is shown in the figure. Find the height h_f to which the ball rebounds. τ is an amount of time.



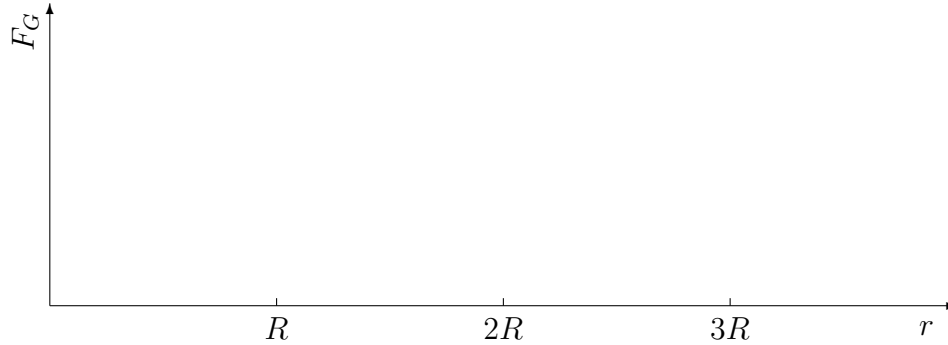
2. (30 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You also work with carts that incorporate a spring that can be compressed and released during a collision, imparting the energy stored in the spring to the carts rebounding from the collision.

You set up the collision with a cart with mass $2m$ with initial velocity $v_{2i} = v$ heading toward a cart with mass m that starts at rest. You measure the final velocity of the cart with mass m in three different experiments, obtaining $v_{1f} = v$, $v_{1f} = \frac{4}{3}v$, and $v_{1f} = 2v$. Analyze these three experiments and determine which experiments must have had a compressed spring released during the collision.

3. (40 points) Remember how we got the gravitational potential energy mgh : the applied force acting against gravity had a magnitude of mg , and we found the area under the force-versus-distance curve, a rectangle of height mg and base h .

Now we want to generalize this to beyond locations close to the Earth's surface. Take the gravitational force magnitude F_G between two point masses m_1 and m_2 separated by a distance r . We will again look at the area under the force-distance curve.

(a) Sketch a graph of F_G versus r .



Now, according to your sketch, do you do more work in changing r from R to $1.1R$, from $2R$ to $2.1R$, or from $3R$ to $3.1R$?

(b) The convention for gravitational potential energy is to say that it is zero when the masses are infinitely far from each other. So the expression for U_G must become very small as r becomes large. Given this, and the behavior you found in part (a), which of the following is the correct general equation for U_G ? (Only one of the options given is consistent with what you found about U_G .)

(i) $U_G = \frac{1}{2}Gr^2$

(ii) $U_G = m_1m_2r$

(iii) $U_G = \frac{m_1}{m_2}e^{-Gr}$

(iv) $U_G = -\ln Gr$

(v) $U_G = -Gm_1m_2/r$

Show your work checking consistency:

(c) Given your U_G , find the escape speed of an object launched away from Earth. This is the minimum speed necessary to never fall back to Earth under the influence of gravity: You start from r equal to the radius of Earth and speed equal to your escape speed, and end up at r equal to infinity and the object at rest. You can look up data about the Earth to find a numerical result.

(d) Find an equation for the radius r_s for the event horizon of a black hole with mass m . The event horizon marks the point beyond which nothing can return, since it would have to travel faster than light. You find r_s by setting the escape speed equal to the speed of light c .