Assignment 6; Phys 185 Name ______________

All the following questions are about the Earth-Moon system. Use the values

$M_e = 5.98 \times 10^{24}$ kg	(mass of the Earth)
$M_m = 7.36 \times 10^{22}$ kg	(mass of the Moon)
$R_e = 6.37 \times 10^{6}$ m	(radius of the Earth)
$R_m = 1.74 \times 10^{6}$ m	(radius of the Moon)
$T_e = 1$ day	(period of rotation of the Earth around its own axis)
$T_m = 27.3$ days	(period of rotation of the Moon around its own axis)
$d = 3.84 \times 10^{8}$ m	(distance between Earth and Moon)
$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$	(Gravitational constant)

The Moon is tidelocked to the Earth. This means that the tidal interactions with the Earth have slowed the rotation of the Moon down to the point where the Moon always presents the same side to the Earth. Therefore the period of rotation of the Earth and Moon around their common center of mass is equal to T_m.

The Earth and Moon do not have exactly uniform densities, but that is not a bad approximation. Therefore use $\frac{2}{5}MR^2$ for the moment of inertia of a sphere around its own axis.

The Earth and Moon's orbits around their common center of mass are elliptical, not exactly circular. But they're close enough to circles that you can assume that they do uniform circular motion.

Ignore every other object in space other than the Earth and Moon. Also ignore the fact that the axis of rotation of the Earth is slightly tilted. All these will introduce small errors.

You do not have to solve the questions symbolically. But it's much easier for me to catch mistakes if you first work out answers symbolically and put in numbers in the end. So if you want generous partial credit, please use symbols as much as possible.

1. (10 points) Find d_{cm}, the distance between the center of mass of the Earth-Moon system and the center of the Earth.

2. (15 points) Find the moment of inertia I_o for the Earth-Moon system, where both the Earth and Moon revolve around their common center of mass. To do this accurately, you will need to use the "parallel axis theorem," which gives the moment of inertia of an object rotated about an axis that is parallel to an axis that passes through its center of mass:

$$I = I_{cm} + Mr^2$$

where I_{cm} is the moment of inertia about an axis through the center of mass, M is the mass, and r is the distance between the center of mass and the parallel axis of rotation.

Then, using I_o, calculate L_o, the orbital angular momentum due to the Earth and Moon revolving around their center of mass.

3. (5 points) Now calculate L_o again, making the simplifying assumption that the Earth is stationary and the Moon is a point mass that revolves around the Earth in a circle with radius d. Using the more accurate result from the previous question, calculate the percentage error you introduce into L_o if you make the assumption that the Moon revolves around the Earth.

4. (10 points) Calculate L_e, the angular momentum due to the Earth spinning around its own axis, and L_m, the angular momentum due to the Moon spinning around its own axis. Then, also using L_o from the previous questions, calculate L_T, the total angular momentum of the Earth-Moon system. Find what percentage of L_T is due to L_o, L_e, and L_m—and therefore determine which of these you can ignore.

5. (15 points) The period T_m depends on d. Using the approximation that the Moon does uniform circular motion around the Earth, derive an equation expressing how T_m depends on d. Then plug the appropriate numbers into this equation to see if you do, in fact, get the correct result for T_m.

6. (5 points) Due to the tidal interactions that tidelock the Moon, the Earth's rate of rotation around its own axis has also been slowing down over the last few billion years. Due to conservation laws, this means d also changes. So in the past, when the Earth's day T_e' was shorter than T_e ($T_e' < T_e$), would the Earth-Moon distance d' have been larger or smaller than d today?

7. (10 points) At a time in the past when T_e' was 20 hours, what would have been d', the distance between the Earth and Moon?

8. (15 points) Calculate the total energy of the Earth-Moon system today. For the gravitational potential energy, use $U = -Gm_1m_2/d_{12}$. Identify which of the energies are very small compared to the total energy, and are therefore negligible.

9. (15 points) The tidal interactions that cause the spin of the Moon and Earth to slow down are like friction; they cause energy losses that are hard to account for. Calculate E_{loss}, the energy lost to tidal heating between the time when T_e' was 20 hours and today.