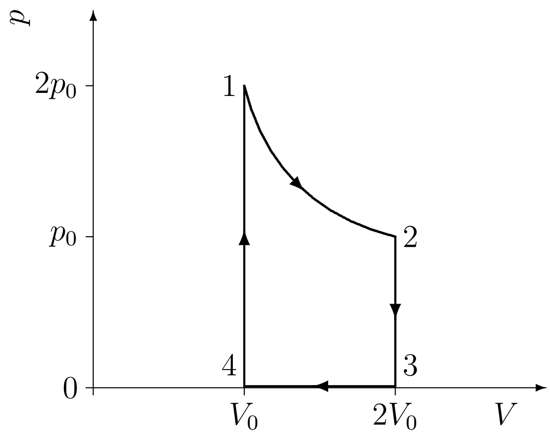
Assignment 7; Phys 185 Name ____________

1. (40 points) It's physically impossible to have a cold reservoir at absolute zero, but let's see what would happen if such a thing were available.

You have a monatomic ideal gas that goes through the cycle $1 \to 2 \to 3 \to 4 \to 1$ shown in the diagram. No gas molecules are added or removed during the cycle.



Find everything (W, ΔU, Q) in terms of p_0 and V_0.

(a) The $1 \to 2$ part of the cycle takes place at *constant temperature*, so $T_1 = T_2$. The area under a constant temperature curve with temperature T on the p-V diagram, going from from an initial V_i to a final V_f, is

$$nRT \ln\left(\frac{V_f}{V_i}\right)$$

Find the work done on the gas for each step of this cycle: $W_{1\to2}$, $W_{2\to3}$, $W_{3\to4}$, $W_{4\to1}$.

(b) Find the change in thermal energy for each step: $\Delta U_{1\to2}$, $\Delta U_{2\to3}$, $\Delta U_{3\to4}$, $\Delta U_{4\to1}$.

(c) Find the heat added to the gas for each step of this cycle: $Q_{1\to 2}$, $Q_{2\to 3}$, $Q_{3\to 4}$, $Q_{4\to 1}$.

(d) Find the total heat input to this gas in one cycle, Q_{in}. Also find the total heat removed from the gas, Q_{out}, and the total work done on the gas, W.

(e) What is the efficiency of this heat engine? (Your result should be a number.)

2. (40 points) If you look up how convection works, you will find $Q/\Delta t = hA\Delta T$, where A is the surface area of an object, and h is a convection coefficient that depends on the material and its geometric shape. You know how conduction and radiation works.

(a) You have two cubes made of identical materials, in identical environments, at identical starting temperatures. Cube 1 has a side of length a, cube 2 has $2a$. Find the ratio of the rates at which each cube cools:

$$\frac{\left(\frac{\Delta T_1}{\Delta t}\right)}{\left(\frac{\Delta T_2}{\Delta t}\right)}$$

Note: ΔT refers to the temperature difference with the environment. ΔT_1 and ΔT_2 are *different*—they refer to the change in temperature over time of cubes 1 and 2.

Hint: Your final result should be a number, with no symbols. Cancel things!

(b) Use this to predict whether in cold climates, small or large animals will have proportionally thicker coats, and area-reducing adaptations such as smaller external ears. Explain.

3. (20 points) Take a small bubble of air at a depth d below the ocean surface. There are n moles of air in the bubble, and air is approximated very well as an ideal gas. Let's assume that the bubble is small enough that we can assume a single depth and a single pressure value accurately characterizes the bubble. Let's also assume that the ocean has a constant temperature T at any depth, and that the air is always in thermal equilibrium with the ocean. Use p_{atm} to represent atmospheric pressure and ρ_w to represent the density of water.

(a) Write down an equation for V, the volume of the bubble.

(b) Now write down an equation for the buoyancy force F_B on the bubble.

(c) Make a rough sketch of the buoyancy force versus depth. Make sure the sketch is clear about whether F_B almost at the surface ($d = 0$) is zero, infinite, or a finite value.