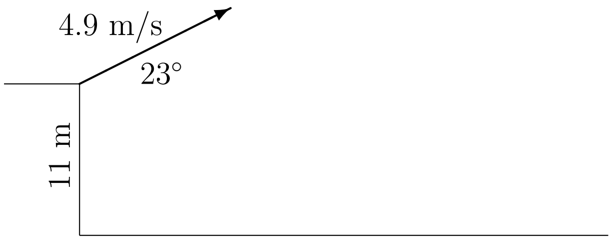
Solutions to Practice 1; PHYS 185

1. (0 points) You're on the surface of the moon, with an acceleration due to gravity of $g' = g/6$. There is no air on the moon, and therefore no air resistance. You stand at the edge of a crater that is 11.0 m deep, and kick a rock into the crater. Say you give the rock an initial speed of 4.9 m/s, and an angle of 23° with the horizontal. At what horizontal distance from the crater edge will the rock hit the bottom of the crater?



Answer: Choose the coordinate origin to be at the launching point. This means the initial values are:

$$x_i = 0 \quad y_i = 0 \quad v_{ix} = v\cos\theta = 4.5 \text{ m/s} \quad v_{iy} = v\sin\theta = 1.9 \text{ m/s}$$

The accelerations are

$$a_x = 0 \qquad a_y = -g' = -1.6 \text{ m/s}^2$$

As the rock hits the bottom of the crater, we know that

$$y_f = -11 \text{ m}$$

We use the equations

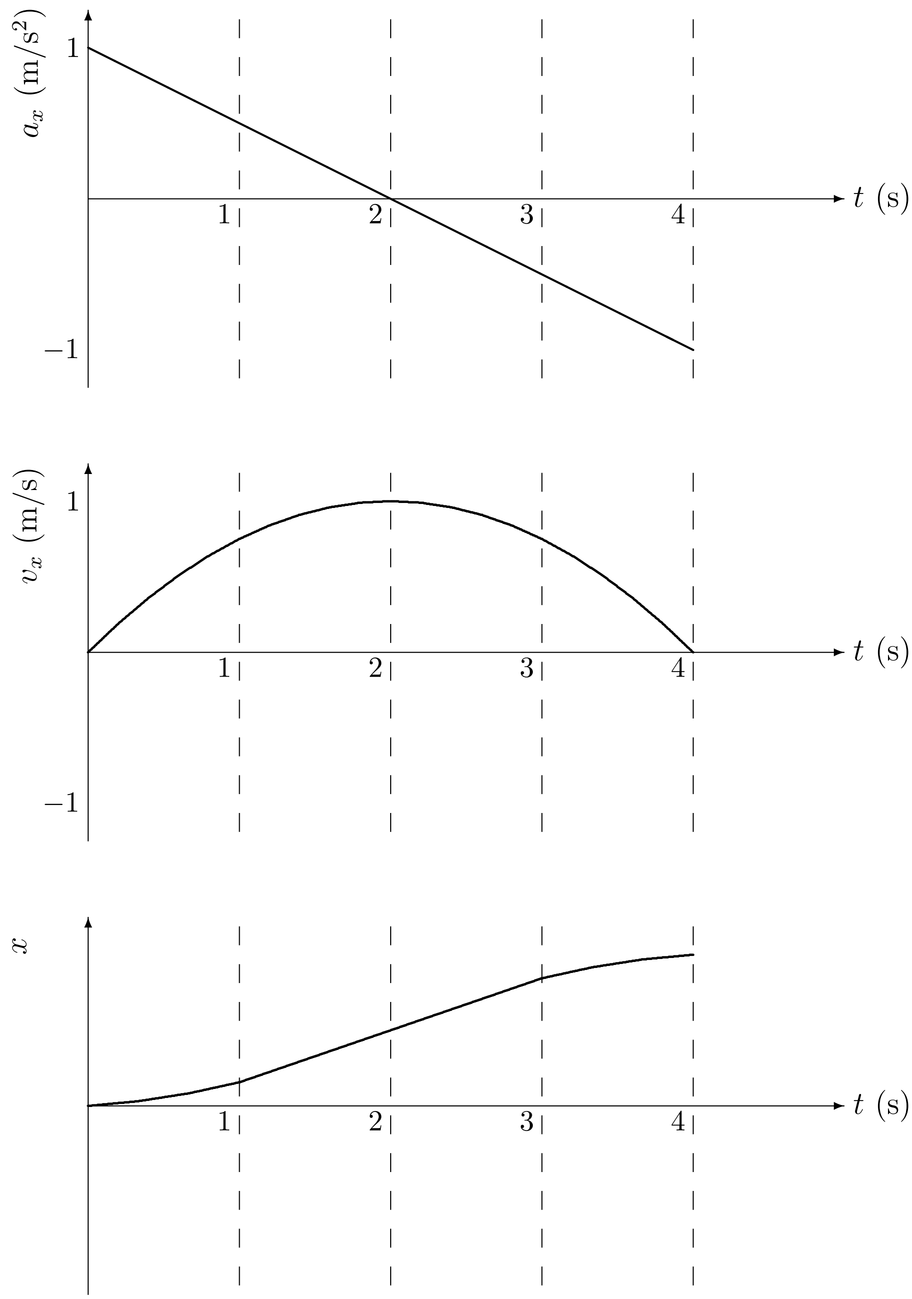
$$x_f = x_i + v_{ix}\Delta t \qquad y_f = y_i + v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

We can use the second equation to solve for Δt. We pick the positive root of the quadratic equation, for $\Delta t = 5.1$ s. Putting this into the equation for x_f, we get

$$x_f = 23 \text{ m}$$

2. (0 points) The top graph displays how acceleration depends on time for an object. Note that the graph is linear, but this is not constant acceleration. Make a sketch of the corresponding velocity versus time and position versus time graphs for this motion, assuming that $v_{xi} = 0$ and $x_i = 0$ for the object at $t = 0$.

Get the numbers right for v_x at $t = 0, 2,$ and 4 s. You don't need exact numbers for the x graph; just sketch the qualitative shape.



3. (0 points) You're doing the experiment in Lab 2, with the cart going up and down an inclined low-friction track. You notice that the accelerations going up and down are slightly different; $a_{\mathsf{up}} = -2.1\ \mathrm{m/s^2}$ and $a_{\mathsf{down}} = -1.9\ \mathrm{m/s^2}$; where these are accelerations along an x-axis tilted to be parallel with the track, with the $+x$-direction pointing up (away from the motion detector).

(a) Find θ, the angle at which the track is tilted. *Hint:* If you solve this symbolically, you'll find that θ depends on $\frac{1}{2}(a_{\mathsf{up}} + a_{\mathsf{down}})$ and g.

Answer: The only difference between the up and down motions is that the kinetic friction force reverses direction, since it points opposite the velocity. The weight and normal forces remain the same. Going up,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta - \mu_k mg\cos\theta = ma_{\mathsf{up}} \qquad \Rightarrow \qquad a_{\mathsf{up}} = -g(\sin\theta + \mu_k\cos\theta)$$

Coming down,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta + \mu_k mg\cos\theta = ma_{\mathsf{down}} \qquad \Rightarrow \qquad a_{\mathsf{down}} = -g(\sin\theta - \mu_k\cos\theta)$$

If we add the equations for a_{up} and a_{down}, we get

$$a_{\mathsf{up}} + a_{\mathsf{down}} = -2g\sin\theta \qquad \Rightarrow \qquad \theta = \sin^{-1}\left[-\left(\frac{a_{\mathsf{up}} + a_{\mathsf{down}}}{2}\right)\frac{1}{g}\right] = 12^\circ$$

(b) Find μ_k, the coefficient of kinetic friction between the track and the cart. *Hint:* If you solve this symbolically, you'll find that μ_k depends on $\frac{1}{2}(a_{\mathsf{up}} - a_{\mathsf{down}})$, g, and θ.

Answer: If we now subtract the equations for a_{up} and a_{down}, we get

$$a_{\mathsf{up}} - a_{\mathsf{down}} = -2\mu_k g\cos\theta \qquad \Rightarrow \qquad \mu_k = -\left(\frac{a_{\mathsf{up}} - a_{\mathsf{down}}}{2}\right)\frac{1}{g\cos\theta} = 0.010$$

This is quite small, as befits a low friction track.

4. (0 points) You have an object with mass m moving on a flat surface, released with initial velocity v_i. The coefficient of kinetic friction between the surface and the mass is μ_k.

(a) Find the distance the object will travel before coming to a halt, in terms of v_i, μ_k, m, and g.

Answer: Adding up the forces will give the acceleration components:

$$\sum F_y = n - mg = ma_y = 0 \qquad \Rightarrow \qquad n = mg$$

$$\sum F_x = -f_k = -\mu_k n = -\mu_k mg = ma_x \qquad \Rightarrow \qquad a_x = -\mu_k g$$

Notice that the mass cancels out. With constant acceleration, we have $v_f = 0$ and

$$0 = v_i - \mu_k g \Delta t \qquad \Rightarrow \qquad \Delta t = \frac{v_i}{\mu_k g}$$

$$\Delta x = v_i \Delta t - \tfrac{1}{2}\mu_k g \left(\Delta t\right)^2 = \frac{v_i^2}{2\mu_k g}$$

(b) You also have a second object which is released with the same initial velocity v_i. This second object is placed in a sleeve that reduces friction by 1%, so that its coefficient of kinetic friction with the surface is $0.99\mu_k$, but increases its total mass by 1%, so that its mass is $1.01m$. Which object will travel a larger distance before coming to a halt?

Answer: The force calculation is exactly the same, including the increased mass canceling out. So we use the same results, only replacing μ_k with $0.99\mu_k$. Therefore

$$a_x = -0.99\mu_k g \qquad \text{and} \qquad \Delta x = \frac{v_i^2}{1.98\mu_k g}$$

This Δx is smaller than the result in (a); the second object travels a larger distance.

5. (0 points) You have two metal spheres which look identical in every respect. One is solid, and the other is hollow, so that the hollow sphere's mass is $m_h = 0.01m_s$. You now do an experiment where you drop these two spheres simultaneously, from the same height.

(a) If you conduct the experiment in a vacuum (in a room where the air has been pumped out), which sphere will hit the floor first? Explain, using forces.

Answer: In a vacuum, the only force on either is their weight. $\sum F_y = -mg = ma_y$, so $a_y = -g$ for both spheres. Their motion will be identical, so they will hit the floor at the same time.

(b) If you conduct the experiment in an ordinary room, with air, which sphere will hit the floor first? Explain, using forces.

Answer: Now there is drag force to contend with. Since both spheres look identical, C_D and A are the same for them. They fall in the same air, so ρ is the same. So at the same velocity, the drag force encountered by both spheres is the same. But since their mass is different, *their accelerations will not be the same.* For the solid sphere,

$$\sum F_y = \frac{1}{2} C_D \rho A v^2 - m_s g = m_s a_y \qquad \Rightarrow \qquad a_y = -g + \left(\frac{1}{m_s}\right) \frac{1}{2} C_D \rho A v^2$$

And for the hollow sphere

$$\sum F_y = \frac{1}{2} C_D \rho A v^2 - m_h g = m_h a_y \qquad \Rightarrow \qquad a_y = -g + \left(\frac{100}{m_s}\right) \frac{1}{2} C_D \rho A v^2$$

The effect of the drag force on the hollow sphere's acceleration is 100 times that of the solid sphere. So it is more affected by air resistance, and it will hit the floor later.