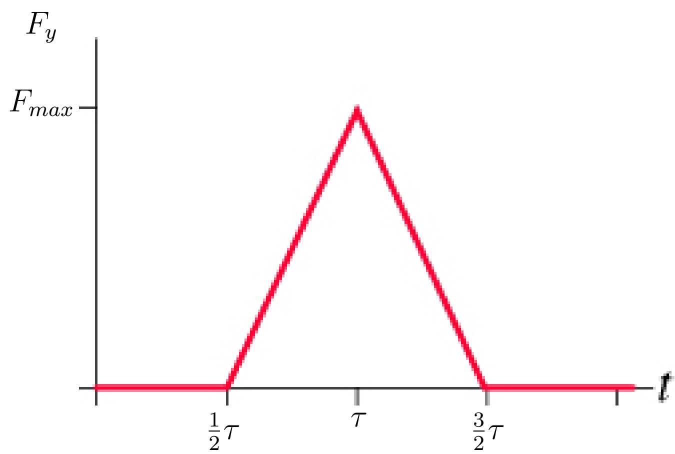
Solutions to Assignment 5; PHYS 185

1. (30 points) A ball with mass m, starting at rest, is dropped from a height of h_i and bounces on a hard floor. The force on the ball from the floor is shown in the figure. Find the height h_f to which the ball rebounds. τ is an amount of time.



Answer: First, the speed of the ball as it hits the floor. From energy conservation, $mgh_i = \frac{1}{2}mv_i^2$. Therefore $v_i = -\sqrt{2gh_i}$, with a $-$ sign because it is headed downward.

This means the momentum of the ball just before hitting the floor is $p_i = mv_i = -m\sqrt{2gh_i}$. The area under the curve is $\Delta p = p_f - p_i$, which is $\Delta p = \frac{1}{2}F_{max}\tau$. So on the bounce back up, the momentum starts from $p_f = p_i + \Delta p = -m\sqrt{2gh_i} + \frac{1}{2}F_{max}\tau$.

On the way up, we start with a velocity of $v_f = p_f/m = -\sqrt{2gh_i} + \frac{1}{2m}F_{max}\tau$. Using energy conservation again, with $\frac{1}{2}mv_f^2 = mgh_f$, we get

$$h_f = \frac{1}{2g}\left(\frac{F_{max}\tau}{2m} - \sqrt{2gh_i}\right)^2$$

2. (30 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You also work with carts that incorporate a spring that can be compressed and released during a collision, imparting the energy stored in the spring to the carts rebounding from the collision.

You set up the collision with a cart with mass $2m$ with initial velocity $v_{2i} = v$ heading toward a cart with mass m that starts at rest. You measure the final velocity of the cart with mass m in three different experiments, obtaining $v_{1f} = v$, $v_{1f} = \frac{4}{3}v$, and $v_{1f} = 2v$. Analyze these three experiments and determine which experiments must have had a compressed spring released during the collision.

Answer: Momentum conservation:

$$(2m)v + 0 = (2m)v_{2f} + mv_{1f} \quad \Rightarrow \quad v_{2f} = v - \frac{1}{2}v_{1f}$$

The change in total energy due to the collision will be the difference in initial and final total kinetic energies, since no relevant potential energies apply, and there is no loss to friction.

$$\Delta E = \frac{1}{2}(2m)v_{2f}^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Putting the result from momentum conservation in there,

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v_{1f}\right)^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Now, we need to investigate the sign of ΔE. If $\Delta E > 0$, extra energy has been added to the carts, which would be because of the spring being released.

The first experiment, with $v_{1f} = v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v\right)^2 + \frac{1}{2}mv^2 - \frac{1}{2}(2m)v^2 = -\frac{1}{4}mv^2 < 0$$

This is an inelastic collision, and since $\Delta E < 0$, there's no evidence for a spring release here.

The second experiment, with $v_{1f} = \frac{4}{3}v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}\frac{4}{3}v\right)^2 + \frac{1}{2}m\left(\frac{4}{3}v\right)^2 - \frac{1}{2}(2m)v^2 = 0$$

This could be an elastic collision, which could happen without a spring being released. So again, there's no evidence for a spring release.

The third experiment, with $v_{1f} = 2v$:

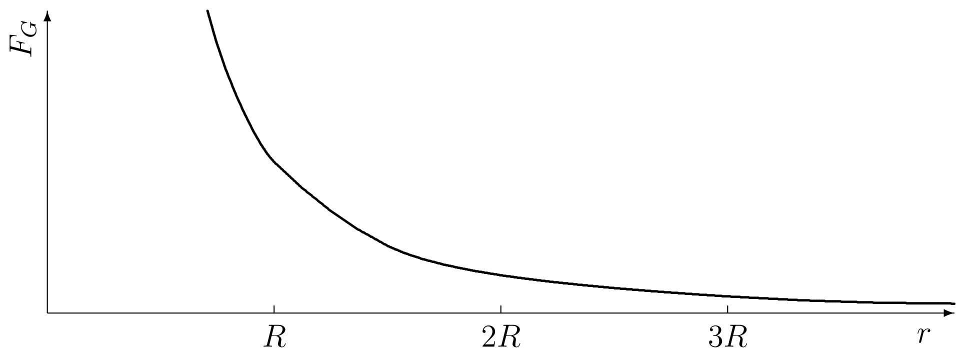
$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}2v\right)^2 + \frac{1}{2}m\,(2v)^2 - \frac{1}{2}(2m)v^2 = mv^2 > 0$$

There is no way $\Delta E > 0$ without the spring release! It must have happened in this third experiment.

3. (40 points) Remember how we got the gravitational potential energy mgh: the applied force acting against gravity had a magnitude of mg, and we found the area under the force-versus-distance curve, a rectangle of height mg and base h.

Now we want to generalize this to beyond locations close to the Earth's surface. Take the gravitational force magnitude F_G between two point masses m_1 and m_2 separated by a distance r. We will again look at the area under the force-distance curve.

(a) Sketch a graph of F_G versus r.



Now, according to your sketch, do you do more work in changing r from R to $1.1R$, from $2R$ to $2.1R$, or from $3R$ to $3.1R$?

Answer: You'll do the most work in going from R to $1.1R$, as that is the largest additional area under the curve. From $3R$ to $3.1R$ is the least work.

(b) The convention for gravitational potential energy is to say that it is zero when the masses are infinitely far from each other. So the expression for U_G must become very small as r becomes large. Given this, and the behavior you found in part (a), which of the following is the correct general equation for U_G? (Only one of the options given is consistent with what you found about U_G.)

(i) $U_G = \frac{1}{2}Gr^2$

(ii) $U_G = m_1 m_2 r$

(iii) $U_G = \frac{m_1}{m_2}e^{-Gr}$

(iv) $U_G = -\ln Gr$

(v) $U_G = -Gm_1m_2/r$**. This is the only one that behaves according to (a), is very small as** r **becomes very large, has the proper units, and where** m_1 **and** m_2 **are interchangeable.**

(c) Given your U_G, find the escape speed of an object launched away from Earth. This is the minimum speed necessary to never fall back to Earth under the influence of gravity: You start from r equal to the radius of Earth and speed equal to your escape speed, and end up at r equal to infinity and the object at rest. You can look up data about the Earth to find a numerical result.

Answer: Use energy conservation. The final energy is 0, since as $r \to \infty$, $U_G \to 0$, and the object is at rest and has no kinetic energy. The initial energy must therefore add up to zero:

$$\frac{1}{2}mv^2 - \frac{Gm_E m}{r_E} = 0$$

Solving for v, we get

$$v = \sqrt{\frac{2Gm_E}{r_E}} = 1.12 \times 10^4 \text{ m/s}$$

(d) Find an equation for the radius r_s for the event horizon of a black hole with mass m. The event horizon marks the point beyond which nothing can return, since it would have to travel faster than light. You find r_s by setting the escape speed equal to the speed of light c.

Answer: You just set

$$\sqrt{\frac{2Gm}{r}} = c \qquad \Rightarrow \qquad r = \frac{2Gm}{c^2}$$