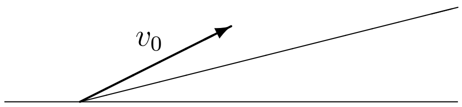
Solutions to Exam 1; Phys 185

1. (40 points) You launch a ball, releasing it with an initial speed v_0 and angle θ with the horizontal. However, you launch the ball not in a flat area, but up the slope of a hill described by a straight line that makes an angle ϕ with the horizontal.



(a) Assume that air resistance (drag) is negligible. Find an equation that describes the location on the hill that the ball will land. The equation should only include known quantities such as v_0, θ, ϕ, and g. You might need math help to describe the hill, or some other things. Ask me—provided you ask me the right questions, I'll make sure you can figure out the math.

Answer: The hill is described by $y = x\tan\phi$, and that is the condition for the ball being on the hillside. Using the x and y-equations for motion with constant acceleration $a_x = 0$ and $a_y = -g$, we get

$$v_{0y}t - \tfrac{1}{2}gt^2 = (v_{0x}t)\tan\phi \quad \Rightarrow \quad v_0\sin\theta t - \tfrac{1}{2}gt^2 = v_0\cos\theta\tan\phi\, t$$

The $t = 0$ solution corresponds to the launch, we want the time for when the ball falls back on the hill:

$$t = \frac{2v_0}{g}\left(\sin\theta - \cos\theta\tan\phi\right)$$

At that time, the location is

$$x = v_{0x}t = \frac{2v_0^2}{g}\left(\sin\theta - \cos\theta\tan\phi\right)\cos\theta$$

$$y = v_{0y}t - \tfrac{1}{2}gt^2 = x\tan\phi = \frac{2v_0^2}{g}\left(\sin\theta - \cos\theta\tan\phi\right)\cos\theta\tan\phi$$

Simplifying is useful, but getting it right is more important.

(b) Check your result: find what the location would be for $\phi = 0$ and for $\theta < \phi$. If your results don't match your notes or don't make sense, you probably have made a mistake in your previous work: go back and fix it.

Answer: When $\phi = 0$, $\tan\phi = 0$. That results in

$$x = \frac{2v_0^2}{g}\sin\theta\cos\theta \qquad y = 0$$

This is the range for a projectile on a flat surface, as you would expect.

When $\theta < \phi$, you will toss the ball into the hill, so you should not get solutions with $x > 0$ and $y > 0$. Look at the factor $(\sin\theta - \cos\theta\tan\phi)$ appearing the solutions. Since $\tan\phi = \sin\phi/\cos\phi$, when $\phi > \theta$,

$$cos\theta \tan\phi = cos\theta \frac{\sin\phi}{\cos\phi} > cos\theta \frac{\sin\theta}{\cos\theta} = \sin\theta$$

This means that

$$\sin\theta - \cos\theta\tan\phi < \sin\theta - \sin\theta = 0$$

In other words, you will get $t < 0, x < 0, y < 0$, indicating no solution further up the hill, as it should be.

(c) You can figure out whether drag is negligible by constructing an inequality that looks like $A \ll B$, where A and B describe quantities you can calculate if you have the numbers, and where the symbol $\ll$ stands for "much less than"; in other words, so small that it is negligible. Construct such an inequality for this case. Feel free to consult me, particularly if you're not sure whether the quantities that go into A and B are easily measurable or not.

Answer: Without drag, the only force on the ball is its weight. So if the drag is always much less than the weight, it will be negligible. So we want

$$\tfrac{1}{2}C_D\rho Av^2 \ll mg$$

The problem is, v is not so easily measurable, and it's variable throughout the flight. We can fix this by using the maximum value of v during its flight. If the drag is very small then, it will always be very small. Notice that v_x^2 never changes, and that for $\phi > 0$, v_{0y}^2 is at its maximum at the point of launch. Since $v^2 = v_x^2 + v_{0y}^2$, the maximum value for v^2 will be v_0^2, and that is a known quantity. The inequality is therefore

$$\tfrac{1}{2}C_D\rho Av_0^2 \ll mg$$

2. (40 points) Lab 3 was not meant to measure g accurately. But you decide to fix some of your sources of error and do a better job. Fist, you devise an automatic release mechanism for the cart, so that it does in fact start with a zero initial velocity as it heads into the first photogate. Then you make some calculations, establishing that the cart never goes fast enough to take the drag force into consideration, and that the pulley's mass and friction at the pulley are so small that they shouldn't matter. And so you set up the equipment and measure Δt and the distance between the photogates, calculating g by using the equation from Lab 3.

(a) Your g result turns out to be improved, but still not good. You then realize that you didn't account for the track possibly being tilted, and that there was friction between the cart and the track. No matter: you realize that if you could measure the tilt angle θ and the coefficient of kinetic friction μ_k, you could still use your data from the experiment, as long as you altered the equation for g to account for the tilt and the friction. Find this improved equation for g.

Answer: With tilted coordinate axes and friction, the force components on the cart are

$$w_x = +m_c g \sin\theta \quad w_y = -m_c g \cos\theta \qquad n_x = 0 \quad n_y = +n$$
$$T_x = +T \quad T_y = 0 \qquad f_{kx} = -\mu_k n \quad f_{ky} = 0$$

where m_c is the cart mass. With m_h, the hanging mass, we get the forces on the . The tension force is still $T = m_h g$. The normal force makes sure $a_y = 0$, so

$$\sum F_y = n - m_c g \cos\theta = 0 \qquad \Rightarrow \qquad n = m_c g \cos\theta$$

$$\sum F_x = m_c g \sin\theta + T - \mu_k n = m_c a_{cx} \qquad \Rightarrow \qquad a_{cx} = (\sin\theta - \mu_k \cos\theta)\, g + \frac{T}{m_c}$$

With m_h the hanging mass, the forces on the hanging mass give

$$\sum F_y = T - m_h g = m_h a_{hy} \qquad \Rightarrow \qquad T = m_h(g + a_{hy})$$

Now, since the cart and the hanging mass are connected, $a_{cx} = -a_{hy} = a$, where a is the theoretical acceleration in Lab 3. Doing the algebra and solving for a, we get

$$a = (\sin\theta - \mu_k \cos\theta)\, g + \frac{m_h(g-a)}{m_c} \qquad \Rightarrow \qquad a = \frac{\left(\sin\theta - \mu_k \cos\theta + \frac{m_h}{m_c}\right)}{1 + \frac{m_h}{m_c}}\, g$$

A form closer to the lab equation is

$$a = \frac{(\sin\theta - \mu_k \cos\theta)\, m_c + m_h}{m_h + m_c}\, g$$

Setting a equal to the experimental acceleration of $2\Delta x/(\Delta t)^2$, we get

$$\frac{(\sin\theta - \mu_k \cos\theta)\, m_c + m_h}{m_h + m_c}\, g = \frac{2\Delta x}{(\Delta t)^2} \qquad \Rightarrow \qquad g = \frac{2\Delta x}{(\Delta t)^2} \frac{(m_h + m_c)}{(\sin\theta - \mu_k \cos\theta)\, m_c + m_h}$$

(b) Check that your answer is the same as what is given in the pre-lab for Lab 3 when $\theta = 0$ and $\mu_k = 0$.

Answer: Since $\sin 0 = 0$ and $\cos 0 = 1$, we end up with

$$g = \frac{2\Delta x}{(\Delta t)^2} \frac{(m_h + m_c)}{m_h}$$

That's what you used in the lab.

(c) Propose an experiment you can perform in the lab with equipment that you've used so far, that will allow you to determine μ_k for the friction between the cart and track. Keep your experiment simple; for example, don't incline the track—that's an unnecessary complication.

Answer: Here is an easy option. Set up a flat track, and release the cart on it, using a motion detector to measure its acceleration. If you send the cart toward the motion detector, the friction and the acceleration will be in the $+x$ direction, so

$$\mu_k n = \mu_k m_c g = m_c a_x \qquad \Rightarrow \qquad \mu_k = \frac{a_x}{g}$$

Note that you should not re-use the set-up for Lab 3, since you want an *independent* handle on g. It would be possible to treat g as an unknown here and put it back into the part (a) equation and solve for g again, but you could not do that if you used your result from (a) to get μ_k in the first place.

3. (20 points) Astronomers observe a planet that has a small moon with a circular orbit. Call the planet's mass M and the moon's mass m; from the orbit, astronomers can also tell that $m \ll M$. Astronomers can also observe r, the distance between the planet and the moon, and T, the period of the moon's orbit around the planet. They also know that gravity is the only significant force between the moon and the planet.

(a) Given all the above information, can the astronomers determine M? If so, find an equation for M.

Answer: Since $M \gg m$, we can take the moon to be in a circular orbit centered on the planet, with radius r. Then,

$$\sum \vec{F} = m\vec{a} \qquad \Rightarrow \qquad G\frac{Mm}{r^2} = m\frac{v^2}{r} = m\frac{(2\pi/T)^2}{r} = \frac{4\pi^2 mr}{T^2}$$

Canceling out the m's and rearranging,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

So the mass of the planet can be inferred from astronomical observations.

(b) Given all the above information, can the astronomers determine m? If so, find an equation for m.

Answer: In the above calculation, m cancels out. So without some other source of information, it won't be possible to figure out the mass of the moon.