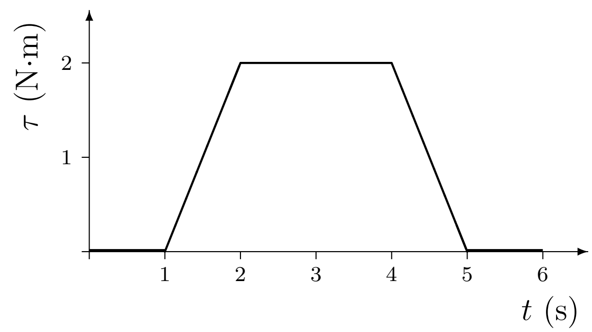
Solutions to Practice 2; PHYS 185

1. (0 points) You decide to spin up a merry-go-round platform from rest by bringing it into contact with a rotating motorcycle wheel. You find that the torque applied by the motorcycle varies in time according to the following graph. The platform is a disk ($I = \frac{1}{2}MR^2$), with radius 1.35 m and mass 118 kg. Assume the platform rotates without friction in its central shaft. Find the angular velocity of the platform after the motorcycle wheel ceases contact.

Note: This is the angular analogue of something you know how to do with *linear* momentum, force, velocity etc. Recall that all we did with force-time curves and change of momentum was just another way of expressing what is in $\sum \vec{F} = \frac{d}{dt}\vec{p}$. The angular analogue of that equation is $\sum \tau = \frac{d}{dt}L$, with L the *angular* momentum. So if you follow the same reasoning with angular quantities, you will get what you want.



Answer: Just like the area under a F-t curve is Δp, the area under this τ-t curve will be ΔL. The area is $\Delta L = 6$ N·m·s. At first, $L_i = I\omega_i = 0$. Therefore

$$L_f = L_i + \Delta L = \Delta L \qquad \Rightarrow \qquad I\omega_f = \Delta L$$

This means

$$\omega_f = \frac{\Delta L}{I} = \frac{2\Delta L}{MR^2} = 0.056 \text{ rad/s}$$

2. (0 points) You have two identical-looking cylinders, with the same mass m and radius r. Cylinder 1 has a moment of inertia $I_1 = mr^2$, while cylinder 2 has $I_2 = kmr^2$, with k an unknown constant. You let them go from rest from the exact same height on an inclined plane, and let them roll without slipping. When they reach the bottom of the incline, you measure their center-of-mass speeds, finding $v_1 = 0.94\, v_2$. What is k?

Answer: Both cylinders start with the same potential energy, and zero kinetic energy. Energy conservation means that the kinetic energies of the cylinders much be the same at the bottom of the incline. We need to account for the rotational kinetic energies, and use $v = \omega r$ for rolling without slipping.

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I_2\omega_2^2$$

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mr^2\left(\frac{v_1}{r}\right)^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kmr^2\left(\frac{v_2}{r}\right)^2$$
$$mv_1^2 = \tfrac{1}{2}(k+1)mv_2^2$$
$$m(0.94\,v_2)^2 = \tfrac{1}{2}(k+1)mv_2^2$$
$$(0.94)^2 = \tfrac{1}{2}(k+1)$$

Which gives

$$k = 2(0.94)^2 - 1 = 0.77$$

3. (0 points) For the following, look up any astronomical data you need.

(a) About where is the axis of rotation of our solar system located? Is this exact, or an approximation? Explain.

Answer: The mass of the Sun is much larger than all the other planets: over a thousand Jupiters. They all rotate about their common center of mass, but the center of mass is almost at the center of the Sun.

(b) Calculate the moments of inertia of the Sun, Earth, and Jupiter around this axis. Be explicit about what approximations you are using to get the moment of inertia for each.

Answer: For the sun, use the moment of inertia for a sphere spinning on an axis through its own center; $I = \frac{2}{5}MR^2$. For all the planets, you will notice from the astronomical data that their distances to the Sun are all much larger than their radii. Therefore the point particle approximation will work for them.

$$I_S = \frac{2}{5}(1.99 \times 10^{30}\ \text{kg})(6.96 \times 10^{8}\ \text{m})^2 = 3.86 \times 10^{47}\ \text{kg} \cdot \text{m}^2$$
$$I_E = (5.98 \times 10^{24}\ \text{kg})(1.50 \times 10^{11}\ \text{m})^2 = 1.35 \times 10^{47}\ \text{kg} \cdot \text{m}^2$$
$$I_J = (1.90 \times 10^{27}\ \text{kg})(7.78 \times 10^{11}\ \text{m})^2 = 1.15 \times 10^{51}\ \text{kg} \cdot \text{m}^2$$

(c) Calculate the angular momentum of the Sun, Earth, and Jupiter. (The Sun's period of rotation around its own axis is 24.5 days.) Add all these angular momenta together to get a total, and state what percentage of this total is associated with each.

Answer: Use $\omega = 2\pi/T$, where T is the period, and $L = I\omega$. 1 year is 3.15×10^7 seconds.

$$L_S = (3.86 \times 10^{47}\ \text{kg} \cdot \text{m}^2)\frac{2\pi}{(2.12 \times 10^{6}\ \text{s})} = 1.14 \times 10^{42}\ \text{kg} \cdot \text{m}^2/\text{s}$$

$$L_E = (1.35 \times 10^{47}\ \text{kg} \cdot \text{m}^2)\frac{2\pi}{(3.15 \times 10^7\ \text{s})} = 2.68 \times 10^{40}\ \text{kg} \cdot \text{m}^2/\text{s}$$

$$L_J = (1.15 \times 10^{51}\ \text{kg} \cdot \text{m}^2)\frac{2\pi}{(11.9)(3.15 \times 10^7\ \text{s})} = 1.93 \times 10^{43}\ \text{kg} \cdot \text{m}^2/\text{s}$$

The total is $L = 2.05 \times 10^{43}$ kg $\cdot$ m^2/s. 5.7% of this belongs to the Sun, 0.13% to Earth, 94% to Jupiter. Most of the angular momentum of the solar system is in the rotation of Jupiter.

4. (0 points) The Earth is in orbit around the sun, and you can calculate its speed v_E at any moment. A comet with mass $m = 0.001 m_E$ and speed v but going in the exact opposite direction approaches the Earth.

(a) Say the comet collides head-on with the Earth. Assume no significant material gets ejected into space. What must v be to produce a 1% change in the speed of the Earth?

Answer: This is a perfectly inelastic collision; the objects stick to each other. Using momentum conservation,

$$m_E v_E - 0.001 m_E v = (m_E + 0.001 m_E)(0.99 v_E)$$

Solving,

$$v = [1 - (1.001)(0.99)]\frac{v_E}{0.001} = 9.01 v_E$$

The speed of the Earth is $2\pi r/T$, where r is the radius of the Earth's orbit, and T is 1 year.

$$v_E = \frac{2\pi(1.50 \times 10^{11}\ \text{m})}{(3.15 \times 10^7\ \text{s})} = 2.99 \times 10^4\ \text{m/s}$$

Therefore

$$v = 2.70 \times 10^5\ \text{m/s}$$

(b) Say the comet misses the Earth entirely, but interacts with Earth's gravity so that it gets a "slingshot" so that it ends up going straight back in the direction it came from. If there is, again, a 1% reduction in the speed of the Earth, what must the initial and final speeds of the comet be?

Answer: This is now a perfectly elastic collision, since there are no dissipative forces like friction involved. Now we have

$$m_E v_E - 0.001 m_E v_i = m_E(0.99 v_E) + 0.001 m_E v_f$$

$$\tfrac{1}{2}m_E v_E^2 + \tfrac{1}{2}0.001 m_E v_i^2 = \tfrac{1}{2}m_E(0.99v_E)^2 + \tfrac{1}{2}0.001 m_E v_f^2$$

With some cancellations,

$$v_E - 0.001v_i = 0.99v_E + 0.001v_f$$

$$v_E^2 + 0.001v_i^2 = (0.99v_E)^2 + 0.001v_f^2$$

The first equation is easier: we get $v_f = 10v_E - v_i$. Substituting that in the second,

$$1000\left(1 - 0.99^2\right)v_E^2 + v_i^2 = (10v_E - v_i)^2 = 100v_E^2 - 20v_E v_i + v_i^2$$

Therefore

$$v_i = \frac{100 - 1000\left(1 - 0.99^2\right)v_E}{20} = 4.00v_E = 1.20 \times 10^5 \text{ m/s}$$

and

$$v_f = 10v_E - v_i = 6.00v_E = 1.79 \times 10^5 \text{ m/s}$$