Solutions to Exam 2; Phys 185

1. (20 points) You're in a spaceship in a circular orbit around a planet. Your distance to the planet's center is r. The captain of your spaceship decides to take a closer look, and so she maneuvers your ship to a distance of $r/2$ to the planet's center. (It's still well clear of the planet's surface.) During the movement to get closer, the only forces applied on the spaceship are radial in direction, with zero tangential component. (Radial means straight inward or outward; tangential is tangent to a circle centered on the planet.) The maneuvers use a negligible amount of fuel, so that m_s, the mass of the ship, remains constant. And when you reach $r/2$, the spaceship has a radial velocity component of zero, just as it was at r.

You have three possibilities for when you reach $r/2$:

Just right. You have a circular orbit with radius $r/2$, without having to do anything else.

Too fast. You need to slow down the spaceship to remain in a circular orbit with radius $r/2$.

Too slow. You need to speed up the spaceship to remain in a circular orbit with radius $r/2$.

Which one is correct? Produce a calculation that shows why.

Answer: $\sum \vec{F} = m_s \vec{a}$ with gravity and uniform circular motion gives

$$G\frac{m_p m_s}{r^2} = m_s \omega_i^2 r \qquad \Rightarrow \qquad \omega_i = \sqrt{\frac{Gm_p}{r^3}}$$

When changing orbit, all the forces on the ship are radial: directed toward the axis of rotation. Therefore none of these forces produce any torque. Gravity is radial, which also does not produce torque. The angular momentum of the spaceship will be conserved. $L = I\omega$, and since the size of the starship is much smaller than r, we can approximate it as a point mass, with $I = m_s r^2$. Setting the initial and final angular momenta equal,

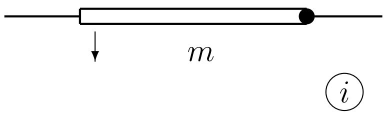
$$\left(m_s r^2\right)\omega_i = \left[m_s\left(\frac{r}{2}\right)^2\right]\omega_f \qquad \Rightarrow \qquad \omega_f = 4\omega_i$$

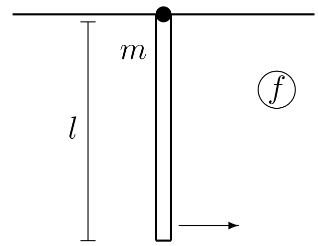
Now, for uniform circular motion with radius $r/2$, the angular velocity must be

$$\omega_f = \sqrt{\frac{Gm_p}{(r/2)^3}} = 2^{3/2}\sqrt{\frac{Gm_p}{r^3}} = 2^{3/2}\omega_i$$

Since $4 = 2^2 > 2^{3/2}$, the spaceship will be going too fast to have a circular orbit. Doing nothing, it would settle into an elliptical orbit. To have a circle, it must slow down.

2. (50 points) You have a uniform thin rod attached to the ceiling at one end. Starting from the rod being up against the ceiling, at rest, you let the rod go, and it swings down. The mass of the rod is m, and its length is l. The effects of the drag force on the rod, and the friction at the ceiling, are negligible. In the following questions, when a variable has a subscript f, it refers to what is happening at the bottom of the arc of the rod's swing, when it is positioned completely vertically.





(a) The moment of inertia of a rod rotating around its center of mass is $ml^2/12$. What is its moment of inertia when the axis of rotation is the point of attachment to the ceiling?

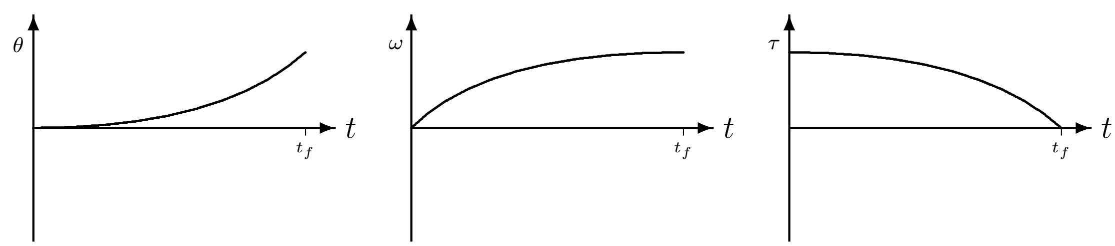
Answer: The parallel axis theorem gives $I = ml^2/12 + m(l/2)^2 = ml^2/3$.

(b) Say your y-axis is pointing upward, and the ceiling height is $y = 0$, so that the initial gravitational potential energy is $U_i = 0$. Circle the potential energy of the rod in position f, as it is vertical. Then provide a reason for your choice.

$$-mgl \qquad -\tfrac{1}{2}mgl \qquad -\tfrac{1}{3}mgl \qquad -\tfrac{1}{6}mgl \qquad -\tfrac{1}{12}mgl$$

Answer: The center of mass of the rod is at its middle, a distance $l/2$ lower than the ceiling. Therefore, $U_f = -\frac{1}{2}mgl$.

(c) Make qualitative graphs of θ, the angle of the rod with the ceiling; ω, its angular velocity; and τ, the total torque on the rod. Give the values of θ and τ when $t = t_i = 0$ and $t = t_f$, the time when the rod is vertical. Briefly explain your reasoning.



Answer: The angle with the ceiling starts from 0 and goes to $\pi/2$. Because of the counterclockwise rotation, ω is positive, and increasingly so; $\omega = \frac{d}{dt}\theta$. And then, the torque $\tau \propto \alpha = \frac{d}{dt}\omega$. τ is due to the $\perp$ component of the rod's weight, which decreases in magnitude as the pencil falls. At the time of release, $\tau = mgl/2$; at t_f, $\tau = 0$.

(d) Can you use linear momentum conservation to find out the angular velocity ω_f? If so, calculate ω_f. If not, explain why.

Answer: No. The rod has a non-zero external total force on it: its weight and the forces at the point of rotation (normal force) do not cancel out. You can see this because the center of mass of the rod accelerates; therefore, $\sum \vec{F} \neq 0$.

(e) Can you use energy conservation to find ω_f? If so, calculate ω_f. If not, explain why.

Answer: Yes. With negligible drag and friction, $E_{\text{loss}} = 0$. Therefore,

$$0 + 0 = \tfrac{1}{2}(\tfrac{1}{3}ml^2)\omega_f^2 - \tfrac{1}{2}mgl \qquad \Rightarrow \qquad \omega_f = \sqrt{\frac{3g}{l}}$$

(f) Can you use angular momentum conservation to find ω_f? If so, calculate ω_f. If not, explain why.

Answer: No. The rod has a non-zero external total torque on it due to its weight. The forces at the point of contact at the ceiling act on the axis of rotation, and therefore produce no torque.

3. (30 points) You set up Lab 6, with carts having masses $m_1 = 0.500\,\text{kg}$ and $m_2 = 1.500\,\text{kg}$. You send cart 1 with an initial velocity of $0.800\,\text{m/s}$ and initial position $x_1 = 0.00\,\text{m}$ toward cart 2, at rest at $x_2 = 1.00\text{m}$. You set the carts so that they bounce off each other magnetically, for an elastic collision. The track is exactly level, the drag force is negligible, and from a previous experiment, you know that $\mu_k g = 0.040\,\text{m/s}^2$ for friction between the carts and the track. Your motion detector tells you that cart 1 reaches $x_1 = 1.00\,\text{m}$ before bouncing back, and that the distance over which the collision takes place is negligibly small.

(a) What is the velocity by which cart 1 enters the collision?

Answer: Call the initial velocity v_0, the velocity going into the collision v_c, and the distance to the collision d. Conservation of energy, accounting for the energy lost to friction, gives

$$\tfrac{1}{2}m_1v_0^2 = \tfrac{1}{2}m_1v_c^2 + \mu_k m_1 g d \qquad \Rightarrow \qquad v_c = \sqrt{v_0^2 - 2d\mu_k g} = 0.748\,\text{m/s}$$

As in Lab 7, we're using $E_{\text{loss}} = -W_f$.

(b) What is the velocity by which cart 1 exits the collision?

Answer: Momentum conservation:

$$m_1v_c + 0 = m_1v_{1f} + 3m_1v_{2f} \qquad \Rightarrow \qquad v_{1f} = v_c - 3v_{2f}$$

Energy conservation:

$$\tfrac{1}{2}m_1 v_c^2 = \tfrac{1}{2}m_1(v_c - 3v_{2f})^2 + \tfrac{1}{2}3m_1 v_{2f}^2 \quad \Rightarrow \quad v_{2f} = \tfrac{1}{2}v_c$$

This means that

$$v_{1f} = -\tfrac{1}{2}v_c = -0.374\,\mathrm{m/s}$$

Since the collision is elastic and takes place over a very small distance, $E_{\text{loss}} = 0$.

(c) What do you predict the motion detector will show for the velocity of cart 1 when the cart comes back to its starting point, $x_1 = 0.00\,\mathrm{m}$?

Answer: Accounting for the loss to friction on the way back, and with v_e the velocity of cart 1 at the end,

$$\tfrac{1}{2}m_1 v_{1f}^2 = \tfrac{1}{2}m_1 v_e^2 + \mu_k m_1 g d \qquad \Rightarrow \qquad v_e = -\sqrt{v_{1f}^2 - 2d\mu_k g} = -0.245\,\mathrm{m/s}$$