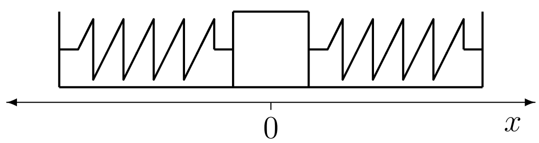
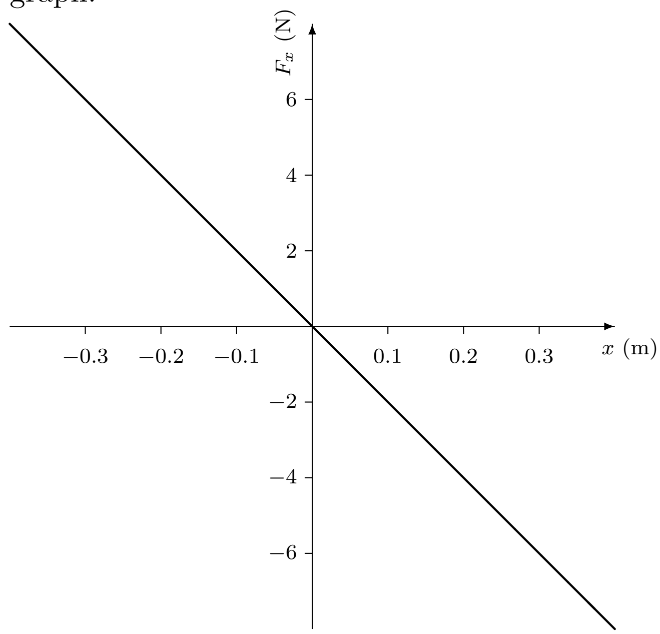
# Solutions to Assignment 1; Phys 186

**1. (30 points)** You have a 0.32 kg object attached to two identical springs, each with spring constant 10.0 N/m, at each end as shown in the diagram. The mass oscillates back and forth horizontally on a frictionless surface.




(a) Using the axes given, draw a graph of the total force on the mass $F_x$ vs. the displacement from equilibrium $x$. Take care to indicate the force direction with appropriately positive or negative quantities. Write in the appropriate numbers on the tick marks on the $F_x$ axis on your graph.




(b) The oscillations of a single spring have an angular frequency $\omega = \sqrt{k/m}$. What is the angular frequency for the oscillations of this double spring setup? Explain.

**Answer:** From the graph you can see that this setup is equivalent to a single spring with an effective spring constant $2k = 20.0$ N/m. Therefore

$$\omega = \sqrt{\frac{2k}{m}} = 7.91 \text{ Hz}$$

**2. (20 points)** You're given a spring, a known mass $m_0$, and an unknown mass $m_1$. The only measuring device you have is a stopwatch. Describe an experiment you would design in order to determine $m_1$. Provide an equation that expresses $m_1$ in terms of $m_0$ and quantities you can measure with your stopwatch.

**Answer:** With a stopwatch, the only thing about the spiring-and-mass systems we can measure is the periods.

The period of oscillations will depend on the mass. So we first measure the period $T_0$ for oscillations with the known mass. (To reduce error, we probably should measure how long $N \geq 100$ oscillations take, and find the period by dividing the reading on the stopwatch by $N$.) We use the same procedure to measure $T_1$. Then, notice that the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}$$

Therefore, $k$ will cancel out if we take the ratio of the two periods:

$$\frac{T_1}{T_0} = \sqrt{\frac{m_1}{m_2}} \quad \Rightarrow \quad m_1 = \left(\frac{T_1}{T_0}\right)^2 m_0$$

**3. (20 points)** We characterize waves by their frequency, wavelength, and amplitude. Audible sound has a frequency range of 20 Hz to 20 kHz, wavelengths between 1.7 cm and 17 m, and a minimum intensity of $10^{-12}$ W/m$^2$. *Ultrasound* can be used for medical imaging, where it can resolve structures with sizes considerably less than 1 cm. The "ultra" in ultrasound must therefore refer to higher than audible frequency, wavelength, or amplitude—which one? Explain. For the range of values in question, you can take the speed of sound to be constant.

**Answer:** Since medical imaging should see detail with sizes less than 1 cm, we need sound with wavelengths *less* than audible wavelengths which start at 1.7 cm. Since $v = \lambda f$, and $v$ is constant, that means that wavelength and frequency are inversely related. So smaller than audible wavelengths corresponds to *higher than audible frequency.* Amplitude is irrelevant.

**4. (30 points)** You have a radio beacon set up in outer space, broadcasting with equal and constant power in all directions.

(a) *Qualitatively* sketch the following graphs of the broadcast waves' intensity, amplitude, frequency, and wave speed vs. the distance $r$ from the beacon.

(b) Give the $r$-dependence of all four variables. The answer for each should be one of $r^2$, $r^1$, $r^0$ (constant), $r^{-1}$, or $r^{-2}$. (*Note:* the symbol "$\propto$" means "proportional to.")

**Answer:**

$$I \propto r^{-2}$$

$$A \propto r^{-1} \text{ since } I \propto A^2$$

$$f \propto r^0$$

$$v \propto r^0$$

with corresponding graphs.