

1. (40 points) An important discovery in the 1980s was the W^+ and W^- , which are among the particles responsible for the weak nuclear force. W^+ and W^- are antiparticles of each other, and they each have a rest mass of $80.4 \text{ GeV}/c^2$. Say you want to create a W^+ and W^- pair by a head-on collision of an electron and a positron (e^- and e^+) into each other at speeds close to the speed of light. The rest mass of an electron (and a positron, its antiparticle) is $0.511 \text{ MeV}/c^2$. (Remember: $1 \text{ GeV} = 1000 \text{ MeV}$.)

- (a) As observed from the lab frame of reference, the e^- and e^+ head toward each other with equal and opposite velocities in the collision. What is the minimum time dilation factor γ that the e^- and e^+ must have in order to produce enough energy to create a W^+ and W^- pair at rest?

(b) Say that in the lab frame of reference, the electron traveled 30.0 km at a constant speed corresponding to the γ you calculated in (a). How far did it travel in its own frame of reference?

(c) Calculate how long it took for the electron to travel 30.0 km in the lab frame of reference. Then calculate how long this time interval was in the electrons own frame of reference.

2. (30 points) A cosmic ray collision creates a muon (a subatomic particle) near the top of the troposphere, at an altitude of 9000 m. The muon heads straight towards the surface at a speed of $0.998c$.

(a) In the reference frame of a ground observer, what is the muon's initial distance to the surface? What is the time the muon takes to reach the surface?

(b) In the reference frame of the muon, what is the muon's initial distance to the surface? What is the time the muon takes to reach the surface?

(c) When measured at rest in the lab, the average lifetime of a muon is 2.2×10^{-6} s. Given your answers to (a) and (b), would an average muon make it to the surface, or does it have to be an exceptionally long-lived one? Explain.

3. (30 points) Special relativity tells you that time and space are weird. But are they so weird as to allow the following?

(a) Lengths contract in frames of reference that are moving relative to the length in question. Is there a frame of reference where the measured length would become negative? *Hint:* Ask yourself: What sort of γ and v for a frame of reference would allow this? Is this possible?

(b) A pilot of a spaceship traveling close to the speed of light can age slower than those who remain behind, and can therefore time travel into the future. Can she also time travel into the past? *Hint:* Ask yourself: what sort of Δt_0 would the pilot need to travel into the past? What sort of γ and v for a frame of reference would allow this? Is this possible?