Solutions to Practice 1; Phys 186

1. (0 points) You have a mass m attached to a frictionless spring with spring constant k, and you set the mass oscillating. In the following list of variables that might describe the resulting motion, draw a circle around those that depend on the mass m:

amplitude, wavelength, period, phase, diffraction

Sketch a graph of this dependence (with m on the horizontal axis) for each variable you circle.

Answer: The amplitude and phase have to do with initial conditions; they do not depend on the mass. Oscillations are not waves; they have no wavelength. Diffraction also applies to waves only.

The period is $T = 2\pi\sqrt{m/k}$, therefore $T \propto \sqrt{m}$. The graph should show a square root dependence.

2. (0 points) In some fiction, the protagonists get shrunk to the size of insects, and try to talk with unshrunken people. Assuming the shrinking process involves nothing but a scaling down (there's no speeding up or anything), would the shrunken people's speech be understood by those who remain normal? Explain.

Answer: The wavelength of sound emitted depends on the size of the source. Shrunken people would produce sounds at a smaller wavelength and thus higher frequency. If they're shrunken small enough, their high-pitched sounds can go beyond the range of human hearing.

3. (0 points) A large refractive index lens on eyeglasses has $n \approx 1.7$, compared to about 1.5 for ordinary glass. Imagine that you read a website that says:

> You will never find glasses with lenses using a material with $n \geq 2$ due to total internal reflection. For such a large index of refraction, the critical angle beyond which light rays from air cannot enter the glass and are totally reflected back is relatively small. This severely restricts peripheral vision, and thus such materials are not suitable to make lenses for eyeglasses.

Does this seem correct, or is it yet another example of the sort of nonsense you can find on the web? Whatever your answer is, support it with a *quantitative* argument.

Answer: This is nonsense. You cannot have total internal reflection for a light ray going from a small index of refraction medium (air) into a larger index of refraction medium (eyeglasses). Calculate the critical angle, for $\theta_2 = 90°$ in $n_1 \sin\theta_1 = n_2 \sin\theta_2$. This gives a critical angle

$$\theta_1 = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 2$$

There is no angle with a sine of 2 or larger. Every light ray from the air will be able to enter the eyeglass lens.

4. (0 points) You have an electric dipole arranged on the y-axis: a $+q$ charge at $x = 0$, $y = a$ and a $-q$ charge at $x = 0$, $y = 0$. The charges are connected by a rigid rod, so the distance between them never changes.

(a) Find an expression for the total electric field created by this dipole on a point on the x-axis, for $x > 0$. (Find the x- and y-components of this electric field.)

Answer: The magnitudes of the electric fields due to the $+q$ and $-q$ charges are

$$E_+ = \frac{kq}{r^2} \qquad E_- = \frac{kq}{x^2}$$

The distance to the $+q$ charge is $r = \sqrt{x^2 + a^2}$. We now get the x- and y-components, using the angle θ between the x axis and the electric field due to the $+q$ charge:

$$E_{+x} = \frac{kq}{r^2}\cos\theta = \frac{kqx}{r^3} = \frac{kqx}{(x^2 + a^2)^{3/2}} \qquad E_{-x} = -\frac{kq}{x^2}$$

$$E_{+y} = -\frac{kq}{r^2}\sin\theta = -\frac{kqa}{r^3} = -\frac{kqa}{(x^2+a^2)^{3/2}} \qquad E_{-y} = 0$$

Adding the components up, the total electric field is

$$E_x = kq\left[\frac{x}{(x^2+a^2)^{3/2}} - \frac{1}{x^2}\right] \qquad E_y = -\frac{kqa}{(x^2+a^2)^{3/2}}$$

(b) Calculate the total force the first dipole exerts on another dipole further down the x-axis, with the $-q$ charge located at $x = 7a$ and the $+q$ charge at $x = 8a$. Express the x- and y-components of this total force as numbers multiplying kq^2/a^2—find these numbers.

Hint: The numbers you find should be between 10^{-3} and 10^{-4} and positive, for both the x- and y-components of the total force.

Answer: Multiply the charge with the electric field from (a) for both charges in the second dipole, and add all the x- and y-components up separately. We get

$$F_{-x} = -kq^2\left[\frac{7a}{(a^2+49a^2)^{3/2}} - \frac{1}{49a^2}\right] = 6.1\times10^{-4}\,\frac{kq^2}{a^2}$$

$$F_{+x} = kq^2\left[\frac{8a}{(a^2+64a^2)^{3/2}} - \frac{1}{64a^2}\right] = -3.6\times10^{-4}\,\frac{kq^2}{a^2}$$

$$F_x = F_{-x} + F_{+x} = 2.5\times10^{-4}\,\frac{kq^2}{a^2}$$

$$F_{-y} = \frac{kq^2a}{(a^2+49a^2)^{3/2}} = 28.3\times10^{-4}\,\frac{kq^2}{a^2}$$

$$F_{+y} = \frac{kq^2 a}{(a^2 + 64a^2)^{3/2}} = -19.1 \times 10^{-4} \, \frac{kq^2}{a^2}$$

$$F_y = F_{-y} + F_{+y} = 9.2 \times 10^{-4} \, \frac{kq^2}{a^2}$$

5. (0 points) Say you set up two finite metal plates and impose a 4V voltage difference between them, much like you did in your equipotential lines lab. For the following two plate arrangements, make qualitative drawings of the 0V, 1V, 2V, 3V, and 4V equipotential lines. Then add the electric field lines. Be sure to show what happens *outside* the plates.

Answer: For the large but finite capacitor, you will get edge effects: the equipotential lines will curve back slightly toward the the plates, and circle around the back of the plates, where they will be very widely separated. With the second capacitor, where the plate size is smaller than the separation, what you have is very similar to a dipole.