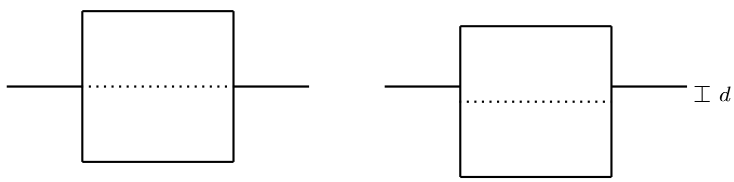
Solutions to Exam 1; Phys 186

1. (25 points) You have a cube with sides a and mass m_c floating in water, with density ρ_w. You mark where the water line is on the cube. Then, you push down the cube by an extra depth d, and let go of the cube. Assume that $d \ll a$ and that you're able to uniformly push the cube down and so forth—in other words, assume that drag forces and any other complications are negligible. You will need to recall how the buoyancy force works.



What is the frequency of oscillation of the cube? (This will be hard to do if you don't talk to me and ask me questions as you are working on this problem!)

Answer: The only forces are the weight of the cube and the buoyancy force. We then use $\sum F_y = F_B - w = m_c a_y$. When the cube was floating, $\sum F_y = 0$, and therefore F_B canceled out w. But when you push the cube down an extra depth d, it picks up extra buoyancy without changing its weight. The extra buoyancy is the weight of water with volume equal to the extra submerged volume:

$$\sum F_y = F_B - w = \rho_w (a^2 d) g = m_c a_y$$

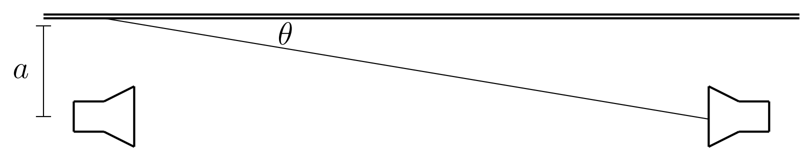
The result is a total force that points directly upward.

With a spring, we had $-kx = ma_x$. With the floating cube, d is a *downward* displacement, so $d = -y$. We can therefore write the equation describing the cube's motion as $-(\rho_w a^2 g) y = m_c a_y$. This is exactly the same as the spring equation, with the constant $\rho_w a^2 g$ playing the role of the spring constant. Since for a spring, $\omega = \sqrt{k/m}$, the angular frequency of the floating cube must be

$$\omega = \sqrt{\frac{\rho_w a^2 g}{m_c}}$$

If you want the frequency, $f = \omega/2\pi$.

2. (25 points) You have the following set up in a lab:

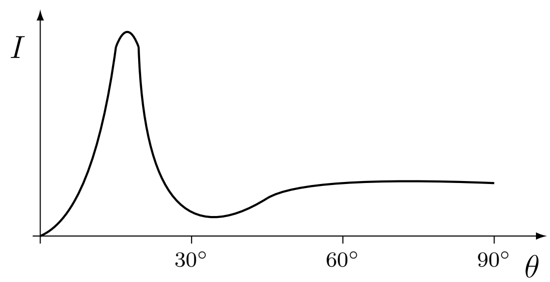


You have a microwave source on the left, and a microwave detector on the right. The microwaves have a wavelength of 3.00 cm. The double lines above represent an oven liner, which reflects the microwaves. The detector can be moved through an angle $0 < \theta < 90°$, as shown. The source is a distance $a = 2.75$ cm to the mirror.

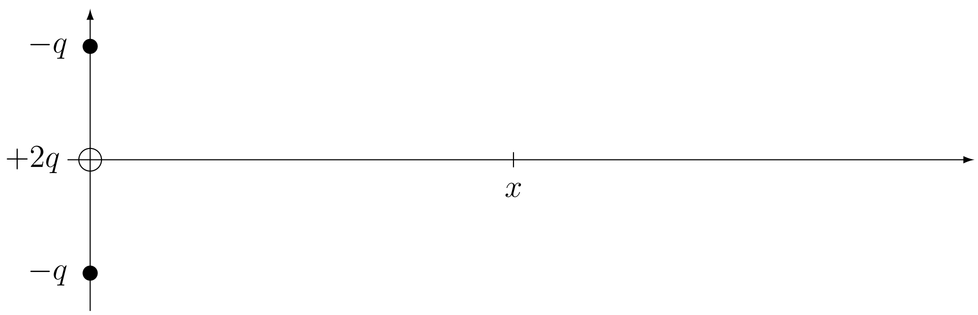
In any textbook, you will see that the effect of a mirror is equivalent to having an identical second source of a light wave located a distance a on the *other side* of the mirror; for example, Figure 25.38 in the online textbook I listed in the syllabus. The oven liner will work the same way with microwaves. And then, there is the extra phase shift to consider, just the same as in Part 2 of your Lab 3.

Sketch a qualitative graph of how the detected intensity will vary with the angle θ, much like Part 3 of your Lab 3. Explain your reasoning. You will be able to find the angles for some maxima or minima of intensity. Calculate these angles. (This will be hard to do if you don't talk to me and ask me questions as you are working on this problem!)

Answer: With the second identical source behind the mirror, this becomes the same situation as a double-slit, with one difference: because of the phase shift with reflection, the locations of the constructive and destructive interference points gets switched. Therefore $\sin\theta_m = m\lambda/d$ will describe the angles of intensity minima, not maxima. The separation between the sources is $d = 2a = 5.50$ cm. That's about the same as in your Lab 3 Part 3; similarly, you will only be able to get angles for $m = 0, \pm 1$. These are $\theta_0 = 0$ and $\theta_{\pm 1} = \pm \sin^{-1}(3/5.5) = \pm 33°$. Sketch the graph for I vs $0 < \theta < 90°$ with I minima at these values. While I will approach zero at $\theta = 0$, because the path lengths become identical, I will not be zero at the $33°$ minimum.



3. (25 points) You have charges $-q$ at $x = 0, y = a$ and $x = 0, y = -a$, and a charge $+2q$ at $x = 0, y = 0$. The positions of the charges are fixed.



(a) Find the total electric field at a point a distance x from the origin on the x axis. In other words, find the total E_x and E_y as functions of k, q, a, and x.

Answer: Label the charges 1, 2, 3, from the top. The magnitudes of the electric fields are

$$E_1 = E_3 = \frac{kq}{a^2 + x^2} \qquad E_2 = \frac{2kq}{x^2}$$

From the symmetry of the set-up, you can see that the y-components of $\vec{E}_1$ and $\vec{E}_3$ will cancel each other out, leaving a total $E_y = 0$. The x-components are, when we include the factors of $\cos\theta = x/\sqrt{a^2 + x^2}$,

$$E_{1x} = E_{3x} = -\frac{kqx}{(a^2 + x^2)^{3/2}} \qquad E_{2x} = \frac{2kq}{x^2}$$

Adding all these up,

$$E_x = 2kq\left(\frac{1}{x^2} - \frac{x}{(a^2 + x^2)^{3/2}}\right)$$

(b) If you can cast your previous answer in a form that puts all the a-dependence in a term that goes like $(1 + a^2/x^2)^{-3/2}$, you can use the approximation

$$\left(1 + \frac{a^2}{x^2}\right)^{-3/2} \approx 1 - \frac{3a^2}{2x^2}$$

valid for $a \ll x$. Use this to show that for large x values, the electric field goes like $E \propto x^{-4}$, an inverse fourth power law. (Ask me for math help if you need it.)

Answer: Rewrite the previous answer as

$$E_x = \frac{2kq}{x^2}\left[1 - \left(1 + \frac{a^2}{x^2}\right)^{-3/2}\right]$$

Use the approximation:

$$E_x \approx \frac{2kq}{x^2}\left[1 - \left(1 - \frac{3a^2}{2x^2}\right)\right] = \frac{3kqa^2}{x^4} \propto x^{-4}$$

By the way, this is a variety of *quadrupole* charge arrangement. Notice that its electric field drops off with distance even faster than with a dipole.

4. (25 points) You have two concentric metal spheres; the inner sphere has a total charge of $-Q$ while the outer sphere has $+2Q$. Draw the electric field lines and equipotential lines all over space: inside the inner sphere, in between the spheres, and outside the outer sphere. To determine the electric field strength, use

$$\text{(Electric flux through a closed surface)} = \frac{1}{\epsilon_0}\text{(Total charge inside that surface)}$$

$$\sum E_{\perp} A = \frac{Q_{\text{in}}}{\epsilon_0}$$

$$\int_{\partial S} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

These equations are all the same thing, in increasing order of mathematical propriety. What you care about is that if $\vec{E}$ is always constant and perpendicular to a closed surface,

$$EA = \frac{Q_{\text{in}}}{\epsilon_0}$$

where E is the outward electric field strength at the enclosing surface you choose, and A is the area of the surface. (I suggest figuring out E before drawing the lines. Check with me to see how you're doing.)

Answer: Since this is a spherically symmetric situation, your enclosing surfaces should be spheres, concentric with the charged spheres. The electric field will then always be perpendicular to the spherical surface, and always have a constant magnitude at the surface. Therefore your electric flux will be $EA = E(4\pi r^2)$, where r is the radius of your enclosing sphere. There are three regions:

Inside the small sphere: In this case, $Q_{\text{in}} = 0$, therefore $E = 0$.

Between the charged spheres: $Q_{\text{in}} = -Q$, therefore $E(4\pi r^2) = -Q/\epsilon_0$ and

$$E = -\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = -k\,\frac{Q}{r^2} \qquad \text{(The $-$ sign means $\vec{E}$ points inward.)}$$

Outside both spheres: $Q_{\text{in}} = -Q + 2Q = Q$, therefore $E(4\pi r^2) = Q/\epsilon_0$ and

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = k\,\frac{Q}{r^2} \qquad \text{(The $+$ sign means $\vec{E}$ points outward.)}$$