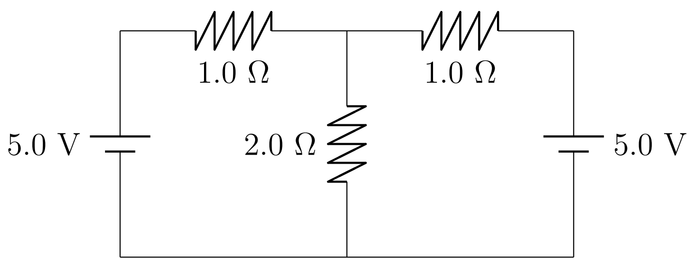
Solutions to Assignment 4; Phys 186

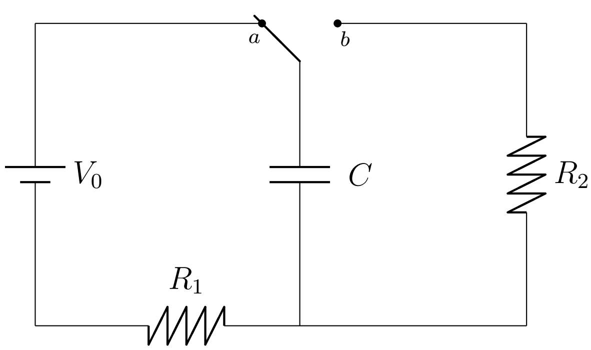
1. (20 points) You have the following circuit. Calculate the voltage across, the current through, and the power dissipated by each resistor.



Answer: Use loops and junctions. Or use this shortcut: the left and right halves of this circuit are identical. So clearly the currents through the 1 Ω resistors are identical: call them I_1. Call the current through the 2 Ω resistance I_2. The junction equation then becomes $I_2 = 2I_1$. Both loop equations are identical: 5 V $= (1\,\Omega)I_1 + (2\,\Omega)I_2$. Putting the equations together, we get 5 V $= (1\,\Omega)I_1 + (2\,\Omega)2I_1$, which means $I_1 = 1$ A. Therefore $I_2 = 2$A.

The voltages: $V_1 = (1\,\Omega)I_1 = 1$ V; $V_2 = (2\,\Omega)I_2 = 4$ V. The powers: $P_1 = V_1 I_1 = 1$ W; $P_2 = V_2 I_2 = 8$ W.

2. (30 points) Here is a simplified (oversimplified) model of a circuit for a camera flash. The resistance R_1 is considerably larger than R_2. When the switch is at a, the capacitor C slowly recharges. When the switch is at b, C rapidly discharges.



(a) Say the switch remains at a for a long time in order to fully charge up the capacitor. This is a "long time" compared to what?

Answer: The time scale for charging up is R_1C—so the time must be long compared to R_1C.

(b) What is the power dissipated by R_2 immediately after the switch is flipped to b? Explain, using this, why a flash requires a small value for R_2.

Answer: Since the capacitor was fully charged, the voltage across it immediately after the switch is flipped will be V_0. (It would not have had any time to discharge yet.) Therefore, using a loop equation, the voltage across the resistor will also be V_0 and the current going through will be V_0/R_2. The power is then

$$P = \frac{V_0^2}{R_2}$$

A flash requires a large burst of energy delivered in a short amount of time. Therefore P should be large—which is why R_2 should be small.

(c) Say $C = 12\ \mu\text{F}$, and $R_2 = 0.21\ \Omega$. How long will it take for the capacitor to discharge 90% of its starting charge?

Answer: The capacitor needs to go down to $1 - 0.9 = 01$ of its original charge. Uisng the exponential discharge relationship,

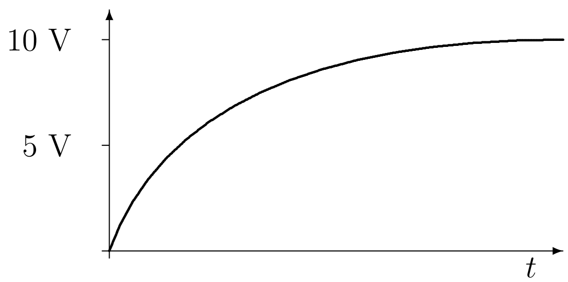
$$Q = Q_0 e^{-t/R_2C} \qquad \Rightarrow \qquad \frac{Q}{Q_0} = 0.1 = e^{-t/R_2C}$$

Therefore

$$t = -R_2C \ln 0.1 = 5.8 \times 10^{-6}\ \text{s}$$

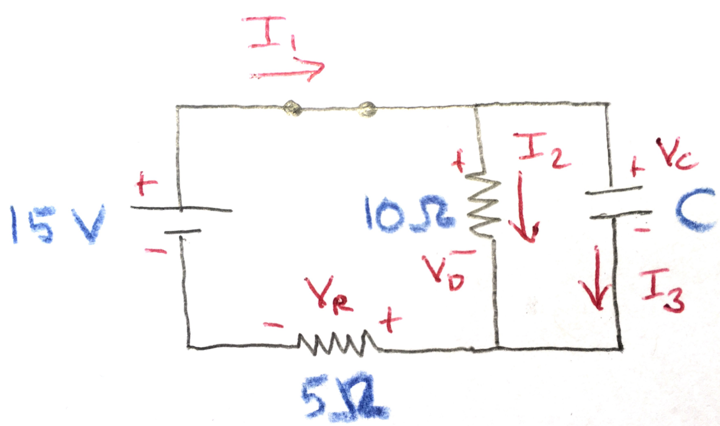
3. (50 points) You have a capacitor (its capacitance is not important), a switch, wires, a 15.0 V DC battery, a 5.0 Ω resistor, and a device that behaves like a 10.0 Ω resistor.

(a) You want the voltage across your device to behave like the following graph after you close the switch; starting at 0.0 V and gradually going up to 10.0 V:



Draw a circuit diagram for the circuit that will do this. Write the junction and loop equations and show that immediately after you close the switch and a long time after you close the switch, the voltage across your device will be 0.0 V and 10.0 V.

Answer: Notice that the voltage graph looks exactly like that for a capacitor charging up. So you should connect your device in parallel with the capacitor, forcing them to have the same voltage. Circuit:

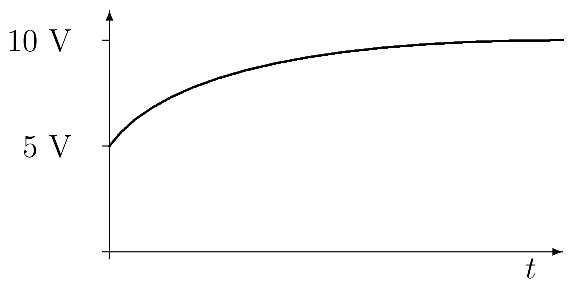


Junction: $I_1 = I_2 + I_3$. Loops: $15\,V = V_D + V_R$ and $V_D = V_C$.

At $t = 0$, the capacitor still has no charge, so $V_C = 0$. Therefore $V_D = 0$ as well, which is what we want.

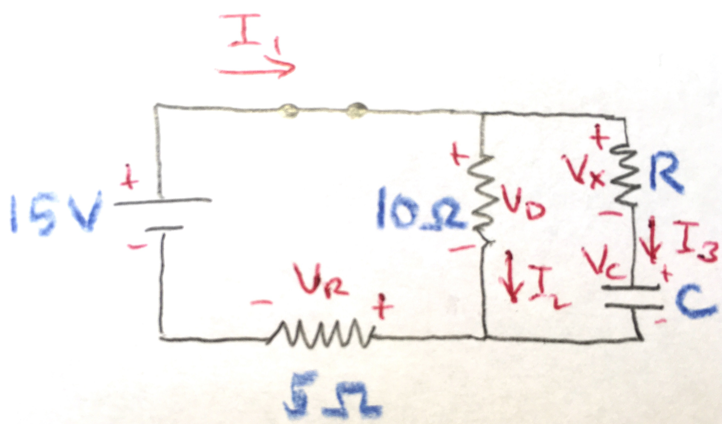
At large times, the capacitor will have fully charged up, so no current will go through it. Therefore $I_3 = 0$. Therefore $I_1 = I_2$ and $15\,V = (10\,\Omega)I_1 + (5\,\Omega)I_1$, which means $I_1 = 1$ A and $V_D = (10\,\Omega)I_1 = 10$ V.

(b) Let's say that instead of the situation in (a), your device requires a voltage graph looking like the following, starting at 5.0 V and gradually going up to 10.0 V:



You can accomplish this by adding an extra resistor R to the circuit that you had for (a). Draw the circuit with the extra resistor R, and use loop and junction equations to calculate the value of R for which the voltage across the device will be 5.0 V immediately after closing the switch and 10.0 V a long time after.

Answer: Circuit:



Junction: $I_1 = I_2 + I_3$. Loops: $15\,V = V_D + V_R$ and $V_D = V_C + V_x$.

At $t = 0$, $V_C = 0$ again, plus we know that $V_D = 5$ V. Therefore $V_x = 5$ V and $V_R = 10$ V. In that case, $I_1 = V_R/(5\,\Omega) = 2$ A, $I_2 = V_D/(10\,\Omega) = 0.5$ A. Using the junction equation, $I_3 = 2 - 0.5 = 1.5$ A. So $R = V_x/I_3 = 5/1.5 = 3.33\,\Omega$.

At large times, no current goes through the capacitor, which is exactly the same situation as in part (a), so as before, $V_D = 10$ V.