Solutions to Exam 2; Phys 186

1. (20 points) You have a positively charged particle moving in a uniform magnetic field pointing in the $+x$ direction. In other words, the vector components $B_x > 0$, $B_y = 0$, $B_z = 0$. The charged particle has velocity components $v_x > 0$, $v_y > 0$, $v_z = 0$.

(a) Draw x and y axes, putting the charged particle at the origin. Then draw arrows indicating the magnetic field the particle experiences, its velocity, and the magnetic force on the particle. If you need to indicate a vector in or out of the paper, use the usual cross or dot notation. Label your vectors.

Answer: $\vec{B}$ toward the right, $\vec{v}$ at an angle in the first quadrant, and $\vec{F}$ into the page.

(b) Pick between these descriptions of the particle's motion. Then explain why it is so.

(i) The particle will move in a circle within the x–y plane

(ii) The particle will move in a spiral with constant radius

(ii) The particle will move with increasing speed in the $+z$ direction

(iv) The particle will oscillate along the x axis

(v) The particle will trace a sine wave along the y axis

Answer: Only the component of the velocity perpendicular to $\vec{B}$ will be affected by the magnetic force. Therefore the parallel component v_x is unaffected, and v_y will be bent into a circle in the y–z plane. This describes a spiral.

2. (25 points) Draw a circuit diagram for an RC-circuit, with an R and C in series, all connected to a function generator set to produce a sine wave. You have the following options for how Q, the charge on the capacitor, behaves as a function of time:

(a) A sine wave with the same frequency as the function generator

(b) A sine wave with double the frequency of the function generator

(c) A square wave with the same frequency as the function generator

(d) A square wave with double the frequency of the function generator

(e) A sawtooth wave with the same frequency as the function generator

(f) A sawtooth wave with double the frequency of the function generator

Sketch qualitative graphs of Q vs t and I vs t, where I is the current in your circuit. Then, using these graphs and any others you care to make, explain why your choice is the correct one. *Note:* You will need to use either your math knowledge, your experience in the lab with waveforms, or playing around with graphing software such as `desmos.com` as an important part of your argument. Make sure you include all the details of your reasoning.

Answer: The circuit is the same as what you had in your Lab 8.

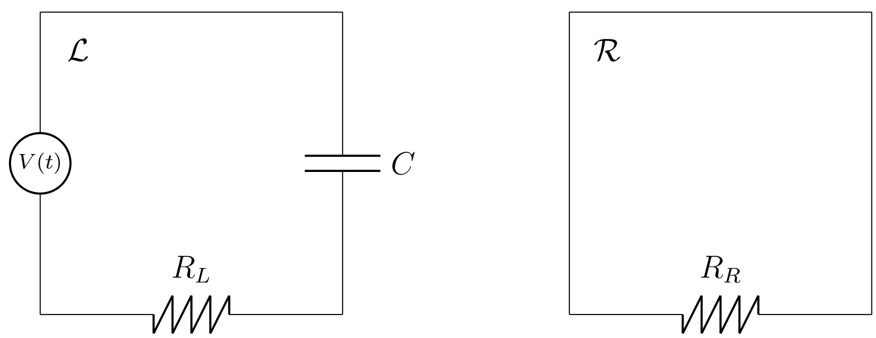
The loop equation gives $V_f = V_R + V_C$. So the voltage across the capacitor and the voltage across the resistor must add up to give the function generator voltage, a sine wave. Putting in the charge and the current,

$$V_f = RI + \frac{Q}{C} = R\frac{d}{dt}Q + \frac{Q}{C}$$

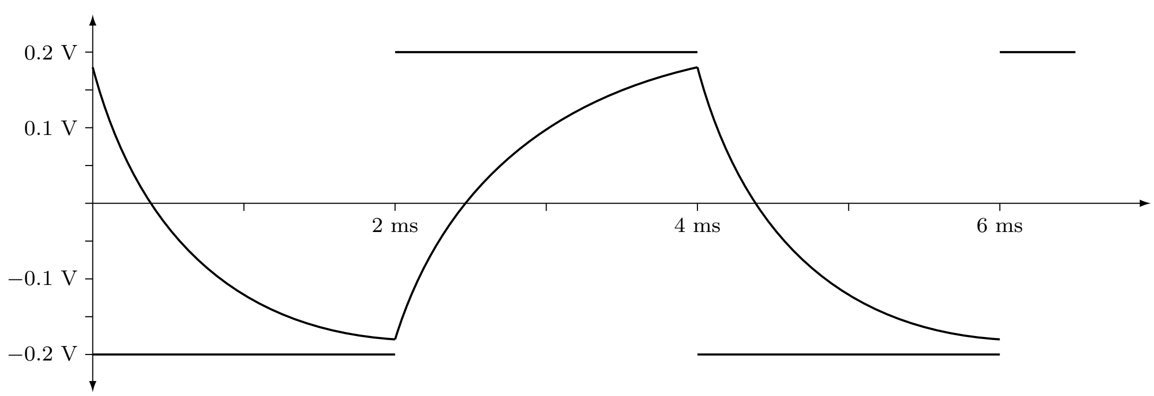
Since the current is due to the charges moving back and forth to the plates of the capacitor, the rate of change of Q is I. Therefore, your sketch of your I graph should look like the slope of your Q graph. The slope of a sine wave is another sine wave, except that it will be phase-shifted by $\pi/2$ (changing a sine into a cosine, for example); in any case, just tracing the slopes should lead you to another sine wave. Now, you might remember from your math that a sine wave plus another sine wave, even when its phase is shifted, is always yet another sine wave with the same frequency. Or, you might remember adding sine waves to get another sine wave while playing around with the oscilloscope in the lap. Failing all that, you can play with adding sine waves on a graphing calculator and see that it's a sine wave that results. In any case, adding a constant times a sine wave plus another constant times a sine wave results in a sine wave with the same frequency, so you can satisfy the loop equation with choice (a).

None of the other choices will work. Working with double the frequency will end up with waves that are double the frequency. The slope of the square wave will give you spikes that would not lead to the original sine wave. And the slopes of the sawtooth will give you a square wave, which again does not add to anything useful.

3. (25 points) You have two circuits next to each other:

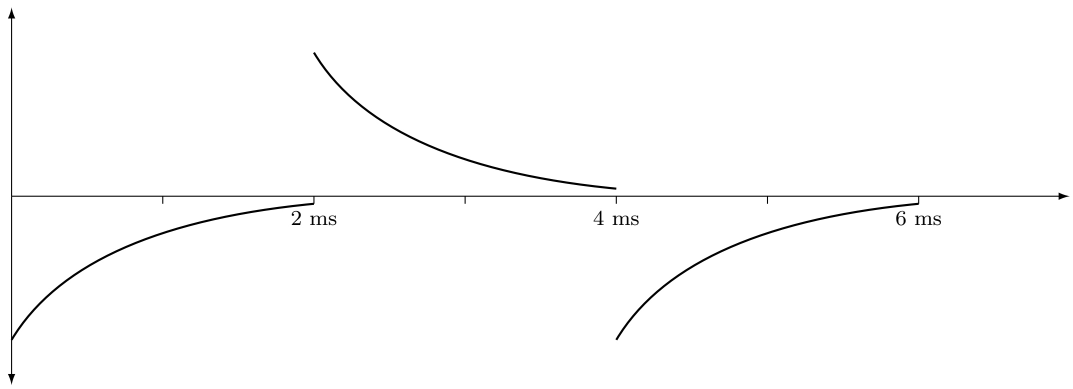


Circuit $\mathcal{L}$ has a voltage source which is a function generator that produces a square waveform $V(t)$, which looks like the following on an oscilloscope:



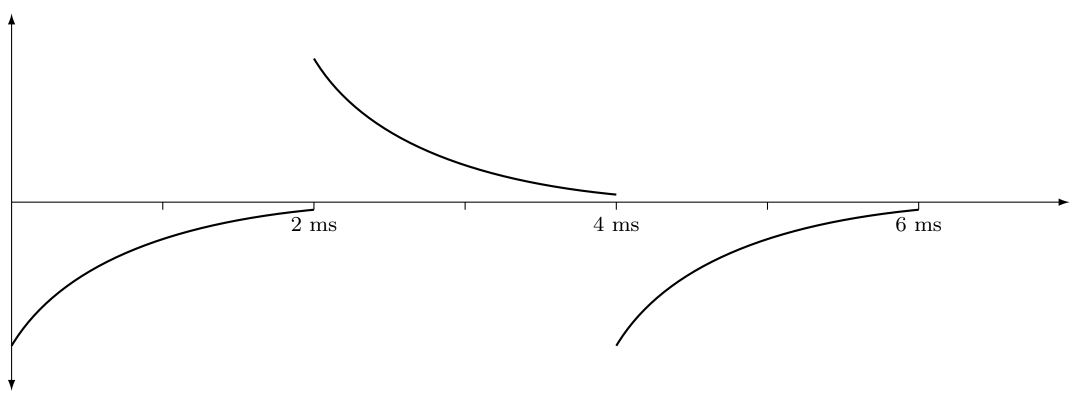
(a) You have the frequency of the function generator set such that the amplitude of the voltage across the capacitor is about 0.18 V (90% of the amplitude of the source voltage). Sketch the shape of the waveform you will see if you measure *the current* in circuit $\mathcal{L}$. Don't put in any numbers—just sketch the waveform. Explain how you arrived at your conclusion.

Answer: I've drawn the capacitor voltage waveform V_c on the upper graph as well. The loop equation for circuit $\mathcal{L}$ is $V(t) = V_c + R_1 I_L$, which means, since the resistance is constant, that $I_L \propto (V(t) - V_c)$. You have to draw the shape of the *difference* between the two voltage waveforms.



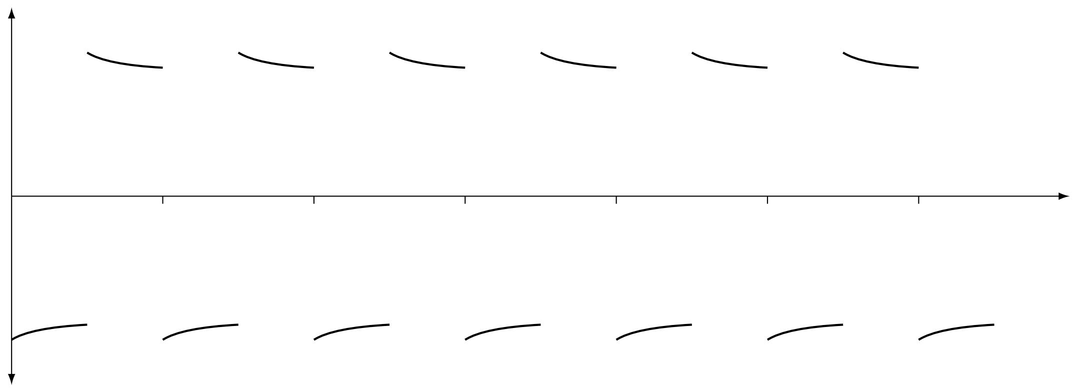
(b) Now sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{R}$ under these conditions. Explain how you arrived at your conclusion.

Answer: I_R is a current induced in circuit $\mathcal{R}$ due to the changing magnetic flux. The magnetic field is produced by I_L. The only thing that is changing is I_L; anything else is a constant that does not affect the shape of the curve. Therefore $I_R \propto -\frac{d}{dt} I_L$. Therefore the shape of this current can be read off the changing slope of the I_L graph above. The graph ends up looking very similar.



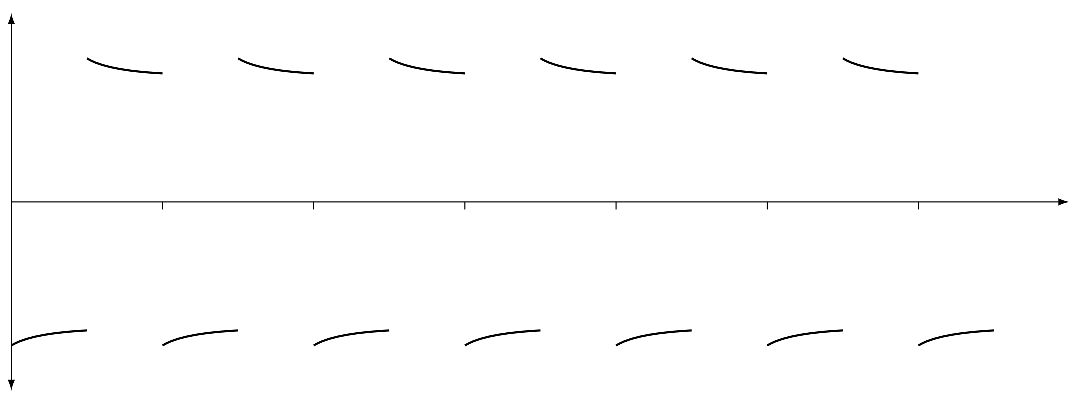
(c) You have the frequency of the function generator set such that the amplitude of the voltage across the capacitor is about 0.02 V (10% of the amplitude of the source voltage). Sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{L}$.

Answer: The reasoning here is very similar. Now the current shape is still the voltage difference, but the frequency is higher and the amplitude of V_c is much smaller.



(d) Now sketch the shape of the waveform you will see if you measure the current in circuit $\mathcal{R}$ under these conditions.

Answer: Again, very similar. You may remember from calculus that the derivative of an exponential is an exponential. Capacitors charging up and discharging are described by exponentials.



4. (30 points) Say you're doing the lab where you accelerated and shot a beam of electrons onto a screen. The mass of an electron is $m_e = 511\,\text{keV}/c^2$.

(a) You accelerated the electrons through a voltage difference of up to 5.00 kV on your dial. At $V_a = 5.00\,\text{kV}$, then, what is the kinetic energy of the electrons in the beam, in units of keV? *Hint:* 1 eV is literally the electron charge magnitude e multiplied by 1 V. Therefore, you shouldn't need any real calculation to get this answer.

Answer: $e(5\,\text{kV}) = 5\,\text{keV}$.

(b) What fraction of the speed of light are these electrons traveling? Use $\frac{1}{2}m_e v^2$ for your kinetic energy, as in your homework and the lab.

Answer: The usual energy conservation gives

$$5\,\text{keV} = \frac{1}{2}(511\,\text{keV})\frac{v^2}{c^2} \quad \Rightarrow \quad \frac{v}{c} = \sqrt{\frac{2\cdot 5}{511}} = 0.140$$

14% of the speed of light. I mentioned that our electrons in the lab were fast.

(c) Recalculate the fraction of the speed of light the electron has, using a more appropriate expression for kinetic energy.

Answer: A more accurate calculation of the speed would go like this. The total energy of the electron is $\gamma m_e c^2$, while the energy of an electron at rest, where $\gamma = 1$, is $m_e c^2$. Therefore, the kinetic energy, which is the additional energy an object has due its motion, must be the difference: $K = (\gamma - 1)m_e c^2$. We can use this relativistic form of kinetic energy to recalculate the fraction of the speed of light the electron has.

Here, γ depends on v/c, so after some algebra,

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{K}{m_e c^2}\right)^2}} = 0.139$$

Again, 14% of the speed of light.

(d) Compare your results in (b) and (c). Do you think relativity was important enough to account for in your lab?

Answer: The answers are very similar; you did not need relativity. While very fast, the electrons are not close enough to the speed of light for relativistic effects to become important.

(e) Say you got a lot more expensive equipment that could provide an accelerating voltage of up to $V_a = 500\,\text{kV}$. In that case, what would you calculate the speed of the electrons to be (as a fraction of the speed of light) if you used $K = \frac{1}{2}m_e v^2$?

Answer: Same calculation:

$$500\,\text{keV} = \frac{1}{2}(511\,\text{keV})\frac{v^2}{c^2} \quad \Rightarrow \quad \frac{v}{c} = \sqrt{\frac{2 \cdot 500}{511}} = 1.40$$

140% of the speed of light. That should raise an eyebrow.

(f) Redo the calculation for v as a fraction of the speed of light with $V_a = 500\,\text{kV}$, but now using the correct expression for kinetic energy.

Answer: Same calculation:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{K}{m_e c^2}\right)^2}} = 0.863$$

86% of the speed of light.

(g) Compare your results with different expressions for kinetic energy in (e) and (f) and interpret what they mean.

Answer: Clearly the nonrelativistic calculation is wrong: faster than light? The electrons are now moving close enough to the speed of light that relativity becomes important, and the proper relativistic kinetic energy expression is necessary.

(h) Again, $V_a = 500\,\mathrm{kV}$. In the lab reference frame, the copper coils with the current providing the magnetic field were circles with a radius of about $R = 6.8\,\mathrm{cm}$. Sketch how the coils look in the electrons' reference frame, and calculate the appropriate dimensions (height, width) for the coil in that frame.

Answer: Here, $\gamma = 1.98$. The coil is not moving in the lab frame, therefore its dimensions are proper lengths. Length contraction will occur along the direction of motion, so the width will contract down to $2 \cdot 6.8/1.98 = 6.9\,\mathrm{cm}$. The height is perpendicular to the direction of motion, so this will not be contracted, remaining at $2 \cdot 6.8 = 13.6\,\mathrm{cm}$. The circle will now look like an ellipse.