Solutions to Practice 3; Phys 186

1. (0 points) You want to conduct an experiment on the interference of sound waves, in a square room that is 4.80 m on all sides. You keep the room empty, and cover the walls, floor, and ceiling with a material that absorbs sound waves without reflecting them back. At the center of one wall of the room, you place two loudspeakers, both of which are capable of emitting pure (sinusoidal) sound waves with varying wavelengths. The volumes of the two loudspeakers are always exactly the same. The distance between the loudspeakers is 0.92 m. At the opposite wall, which is 4.80 m away from the loudspeakers, you place a microphone that measures the sound intensity. You mount the microphone on a track on the wall, so that you can move it to either side from the center of the wall.

(a) Explain why this experiment would not work in an ordinary room, cluttered with objects and with walls that reflect sound.

Answer: In an ordinary room, we would have too many sources of waves, distributed randomly, all interfering with each other. This would wash out any interference effects that would show up with just two sources.

(b) You have one speaker putting out sound with a wavelength of 0.15 m, and the other with a wavelength of 0.26 m. Will you observe a stable (constant over long periods of time) interference pattern with the microphone on the opposite wall? Explain.

Answer: Since the wavelengths are not the same, no stable interference pattern will develop.

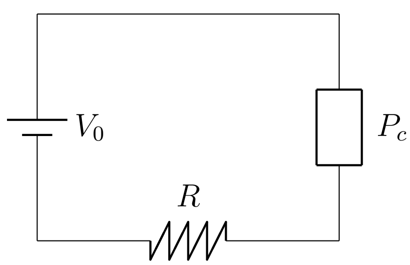
(c) You now have both speakers putting out sound, in phase (peaks emitted at the same time) with the same wavelength of 0.15 m. Sketch a *qualitative* graph of sound intensity on the vertical axis vs. microphone position on the horizontal axis. The position y should vary between -2.4 m and 2.4 m. Calculate the positions along the y-axis where the intensity peaks are located, and indicate how many peaks will be seen from $y = -2.4$ m to 2.4 m.

Answer: This is a double slit experiment, with a characteristic interference pattern. We can calculate the peaks using $\sin\theta = m\lambda/d$, with θ as the angle on a right triangle with sides of y and 4.8 m. Therefore the peak locations are

$$y = (4.8\text{ m})\tan\left(\sin^{-1}\frac{m(0.15\text{ m})}{(0.92\text{ m})}\right)$$

with $m = 0, \pm1, \pm2, \ldots$. The central peak is $m = 0$, giving $y = 0$. For $m = \pm1$, $y = \pm0.79$ m. For $m = \pm2$, $y = \pm1.65$ m. For $m = \pm3$, $y = \pm2.7$ m, which will not be detected. So five peaks will be seen.

2. (0 points) Electric power transmission lines are set up to minimize resistive power losses over long distances. Here is a simplified model of a circuit with a power plant, power lines, and a city consuming power. The power plant is a battery, which supplies a voltage V_0 and puts out a current I_0. The power lines are a fixed resistance R. And we will represent the city as a device that simply consumes a constant power, P_c.



(a) The power supplied by the battery is $P_0 = V_0 I_0$. Show that this is so, using the relationship of voltage to charge q and energy difference ΔU_E, and the relationship of current to charge q and time Δt.

Answer: Power is the rate that energy is extracted from the battery, $P = \frac{d}{dt}U_E$, or in our non-calculus terms, $P = \Delta U_E/\Delta t$. The relationship between voltage and energy is $qV_0 = \Delta U_E$ where q is electric charge. Current is the rate at which charges go by, so $I_0 = q/\Delta t$.

Therefore
$$V_0 I_0 = \left(\frac{\Delta U_E}{q}\right)\left(\frac{q}{\Delta t}\right) = \frac{\Delta U_E}{\Delta t} = P_0$$

(b) In the circuit above, the power supplied by the battery is $P_0 > P_c$. Show that the power lost to dissipation by R becomes smaller as V_0 becomes larger. Hence power lines operate at very high voltages to minimize the loss.

Answer: Energy conservation means that the power dissipated by R and used by the city must be supplied by the battery: $P_0 = P_R + P_c$. It's a circuit with one loop and no junctions, so the same current I_0 goes through r and the battery. Therefore

$$V_0 I_0 = I_0^2 R + P_c$$

To minimize P_R, I_0 has to be as small as possible. But P_c is a constant, and the battery must supply $P_0 > P_c$. This means that lowering I_0 can only happen if V_0 is increased at the same time. So we end up with high voltage power lines.

3. (0 points) If you have a glass of water with a mass m_1 and volume V_1, and another with mass m_2 and volume V_2, and you combine them, you get a total mass of $m_1 + m_2$ and a total volume of $V_1 + V_2$.

Let's define the volume of a black hole as the volume within the sphere with the radius r we calculated in class for the event horizon a black hole of mass m. In that case, let's say we have a black hole with a mass m_1 and volume V_1, and another with mass m_2 and volume V_2, and these black holes merge. Will the total mass of the new black hole be less than, equal, or greater than $m_1 + m_2$? Will the total volume be less than, equal, or greater than $V_1 + V_2$?

Answer: For the event horizon, $r \propto m$, while the volume $V \propto r^3$. The merged black hole will have mass $m_1 + m_2$—there could be some energy lost to gravitational waves, but this will be negligible compared to the black hole masses. But then, the new radius will be $r_1 + r_2 \propto m_1 + m_2$. Therefore the new volume will be proportional to $(r_1+r_2)^3 > r_1^3+r_2^3$. Therefore $V > V_1+V_2$!

4. (0 points) You have a radioactive sample that is a mixture of an α-emitter and a β-emitter. The α-emitting isotope contributes an initial activity of 10.0 counts/second, and has a half-life of 1.0 day. The β-emitting isotope initially contributes 5 counts/second, and has a half-life of 10.0 days.

(a) On the following graph, sketch the α, β, and total activity values vs time. Find exact activities for all three (α, β, total) for days 1, 5, and 10.

Answer: The α graph is a rapid exponential decay, while the β graph is a slower exponential decay. The total activity is the sum of these two exponentials. If $A_{0\alpha}$ and $A_{0\beta}$ refer to the initial activities, this is

$$A_{0\alpha}e^{-(\ln 2)t/t_\alpha} + A_{0\beta}e^{-(\ln 2)t/t_\beta}$$

After day 1: $A_\alpha = A_{0\alpha}2^{-1/1} = 5$ cps, $A_\beta = A_{0\beta}2^{-1/10} = 4.67$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 9.67$ cps.

After day 5: $A_\alpha = A_{0\alpha}2^{-5/1} = 0.32$ cps, $A_\beta = A_{0\beta}2^{-5/10} = 3.54$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 3.86$ cps.

After day 10: $A_\alpha = A_{0\alpha}2^{-10/1} = 0.01$ cps, $A_\beta = A_{0\beta}2^{-10/10} = 2.5$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 2.51$ cps.

(b) The shape of this total activity graph is different than what you would get if you had a pure α or pure β-emitter. How so? Explain using the appropriate math.

Answer: The shape of a graph that is the sum of two exponential decays is different from an exponential decay. In other words, there is no value τ such that $a\,e^{-t/\tau} = b\,e^{-t/\tau_\alpha} + c\,e^{-t/\tau_\beta}$ when $b \neq 0$ and $c \neq 0$.

5. (0 points) Consider a particle with mass m confined to a 1D box, so that it is impossible to find the particle outside $0 \leq x \leq L$. Other than the confinement, the particle is not interacting with anything, so its energy is purely kinetic energy. Now say that you have a particle in the lowest energy level, the ground state, with energy $E_1 = h^2/8mL^2$. This means that you can calculate the particle's momentum:

$$\frac{h^2}{8mL^2} = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{p^2}{2m}$$

Solving for p, we get

$$\sqrt{p^2} = \sqrt{\frac{h^2}{4L^2}} \qquad \Rightarrow \qquad p = \frac{h}{2L}$$

But notice that since h and L are known exactly, therefore p is known exactly. And this means there is no uncertainty in your knowledge of the momentum: $\Delta p = 0$. But since the particle is confined, $\Delta x \approx L$; in any case, $\Delta x < \infty$. Therefore

$$\Delta x \Delta p = 0 < \frac{h}{4\pi}$$

The Uncertainty Principle is violated! But this can't be right. Find the error in the reasoning above. Some options for you to consider:

- Maybe $\Delta x = \infty$ because the particle can quantum tunnel outside the box.
- Maybe h is not known exactly, so its uncertainty needs to be taken into account.
- Maybe $\Delta p > 0$ because there is a subtle mistake in the calculation.

Hint: Remember that momentum is a vector!

Answer: There *is* a mistake in the calculation.

Physically, if the momentum was $p = h/L$, it would mean the particle would be continually moving in the $+x$ direction. But it can't—it hits the wall of the box. The argument overlooks the fact that a square root has two values:

$$\sqrt{\frac{h^2}{L^2}} = \pm\frac{h}{L}$$

In other words, the particle may also be moving in the $-x$ direction. In fact, the probabilities of both directions are equal. Since it is not certain whether the particle has $p = +h/L$ or $p = -h/L$, the uncertainty $\Delta p > 0$.