Solutions to Exam 3; Phys 186

1. (15 points) You build two tall towers on two islands far away from each other, calculating that a perfectly straight line connecting the tops of the towers is blocked by the curving surface of the Earth. You then shine a red laser from one tower, and to your surprise, it is observed from the other. Consider the following three explanations. One is correct, one is correct but the effect size is too small to account for the observation, and one is dead wrong. Identify which one is which, and explain why.

(a) The atmosphere becomes less dense as altitude increases; which means its index of refraction becomes smaller. Therefore the path of the laser beam will curve very slightly.

Answer: Light passing from a lower index of refraction region (high altitude) to a higher index of refraction region (lower in the atmosphere) will be refracted closer to the normal. This means the path of light will curve just above the obstacle presented by the curvature of the Earth, letting the laser beam reach from one tower to the other.

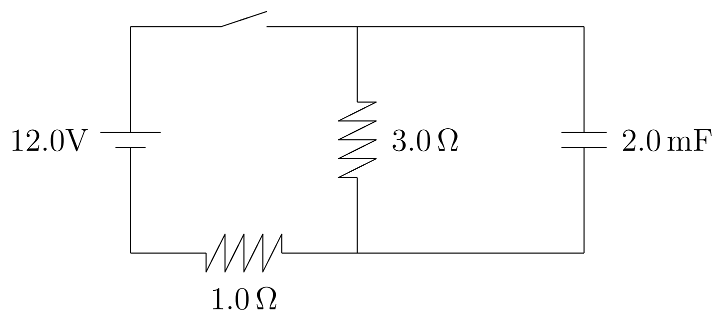
(b) Earth's gravity will affect the light, bending it so that it does not follow an absolutely straight line.

Answer: While this is technically true, the Earth's gravitational field is much too weak to create a noticeable effect. If the towers were close to an Earth-size black hole, the story would be different.

(c) Red light has a larger wavelength than blue, which enhances its constructive interference with the air molecules in the atmosphere.

Answer: This is gibberish.

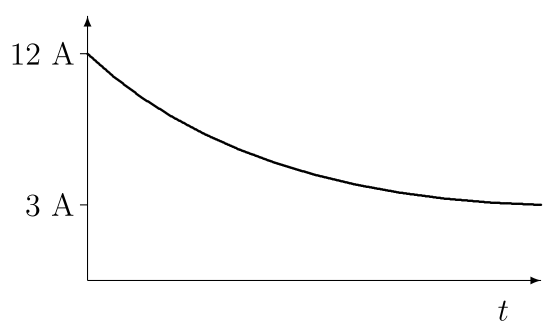
2. (15 points) At time $t = 0$, you close the switch. Sketch a *qualitative* graph of the current through the $1.0\,\Omega$ resistor versus time t. On your graph, indicate the exact numerical value for this current at $t = 0$, immediately after the switch is closed, and the value of the current as $t \to \infty$, after a very long time.



Answer: At $t = 0$, the capacitor starts charging up, and short-circuits the $3\,\Omega$ resistor. We get $I_1 = (12\,\mathrm{V})/(1\,\Omega) = 12\,\mathrm{A}$.

As $t \to \infty$, the capacitor will be fully charged, and we can take it out of the circuit. We will be left with the two resistances in series. The current will be $I_1 = (12\,\mathrm{V})/(1\,\Omega + 3\,\Omega) = 3\,\mathrm{A}$.

In between, the current will decay in a characteristic exponential fashion, but not down to zero.



3. (15 points) You redo the e/m lab (Lab 9), but the wiring is a tangle and you've forgotten what knob controls the electric field strength and what knob is for the magnetic field strength. You play with each and see the following:

- With Knob 1 on but Knob 2 at zero, the electron beam traces a circular arc on your screen.

- With Knob 2 on but Knob 1 at zero, the electron beam traces a parabola.

Which knob controls the electric field strength and which controls the magnetic field? Explain.

Answer: Knob 1 controls the magnetic field. The magnetic field is uniform, which means that the magnetic force is always constant in magnitude and perpendicular to the velocity, which results in a circular path.

Knob 2 controls the electric field. In the experimental setup, the electric force is always perpendicular to the original direction of the beam. This results in motion with constant acceleration, like a projectile under the influence of gravity. Hence a parabola.

4. (20 points) A rough value for the density of interstellar space is one H atom per cubic meter. What is the radius of a black hole with this density? Your result should be large; express it in the most appropriate of these units: (a) the radius of the Earth, (b) an astronomical unit (distance between the Earth and Sun), (c) a light year. Having done so, tell me what you think your result means.

Answer: Looking it up, $m_H = 1.67 \times 10^{-27}\,\mathrm{kg}$. Therefore the density we're looking for is $\rho = 1.67 \times 10^{-27}\,\mathrm{kg/m^3}$.

A black hole has radius $R = 2GM/c^2$, which means $M = (c^2/2G)R$. Its density is

$$\rho = \frac{M}{V} = \frac{c^2 R}{2G}\frac{1}{\frac{4}{3}\pi R^3} = \frac{3c^2}{8\pi G R^2}$$

Solving for the radius and putting in the numbers,

$$R = \sqrt{\frac{3c^2}{8\pi G\rho}} = 3.1 \times 10^{26}\,\mathrm{m}$$

This is enormous; we need to use light years. Converting,

$$R = 3.28 \times 10^{10}\, c \cdot \mathrm{years}$$

This is an appreciable fraction of the observable universe! But then again, a black hole is what light can't escape from, and light can't escape from our universe either.

5. (15 points) You have a radioactive sample that is a mixture of an α-emitter and a β-emitter. The α-emitting isotope contributes an initial activity of 8.0 counts/second, and has a half-life of 1.0 day. The β-emitting isotope initially contributes 7 counts/second, and has a half-life of 10.0 days.

(a) On the following graph, sketch the α, β, and total activity values vs time. Find exact activities for all three (α, β, total) for days 1, 5, and 10.

Answer: The α graph is a rapid exponential decay, while the β graph is a slower exponential decay. The total activity is the sum of these two exponentials. If $A_{0\alpha}$ and $A_{0\beta}$ refer to the initial activities, this is

$$A_{0\alpha}\, 2^{-t/t_\alpha} + A_{0\beta}\, 2^{-t/t_\beta}$$

After day 1: $A_\alpha = A_{0\alpha}\, 2^{-1/1} = 4$ cps, $A_\beta = A_{0\beta}\, 2^{-1/10} = 6.53$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 10.53$ cps.

After day 5: $A_\alpha = A_{0\alpha}\, 2^{-5/1} = 0.25$ cps, $A_\beta = A_{0\beta}\, 2^{-5/10} = 4.95$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 5.20$ cps.

After day 10: $A_\alpha = A_{0\alpha}\, 2^{-10/1} = 0.0078$ cps, $A_\beta = A_{0\beta}\, 2^{-10/10} = 3.50$ cps, $A_{\alpha+\beta} = A_\alpha + A_\beta = 3.51$ cps.

(b) Let's say you were given the sample and could only determine the initial total activity at day 0. What quick and easy experiment could you do to determine what the contribution to the activity was due to α and what due to β?

Answer: As I mentioned in your lab 9, α penetrates next to nothing, while β penetrates thin barriers quite well. So put a thin sheet of paper in front of the sample, and the activity you will then measure will be very close to $A_{0\beta}$.

6. (20 points) You have a very small particle under the influence of a force that acts exactly like a spring force with constant k. Let x stand for the distance from equilibrium, and $p = mv$ the momentum of the particle. The particle speed $v \ll c$.

(a) Write down the total energy of the particle as a function of x and p, with m and k as constants.

Answer: The nonrelativistic kinetic energy plus the potential energy is

$$E_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2m}p^2 + \tfrac{1}{2}kx^2$$

where you can use $v = p/m$ for the nonrelativistic momentum.

(b) If this was a classical (non-quantum) particle, would its minimum total energy be zero or non-zero? Explain.

Answer: The minimum energy is zero, which corresponds to a motionless ($p = 0$) particle at the equilibrium point ($x = 0$).

(c) If this was a quantum particle, would its minimum total energy be zero or non-zero? Explain.

Answer: If the particle had $p = 0$ and $x = 0$, we would be completely certain about its momentum and location: $\Delta p = 0$ and $\Delta x = 0$. This would then mean that $\Delta x \Delta p = 0 < h/4\pi$, which violates the uncertainty principle.

The particle will instead have a probability distribution for its location and momentum, with non-zero standard deviations. And since it will not be stationary and at equilibrium, its total minimum energy will be larger than zero.