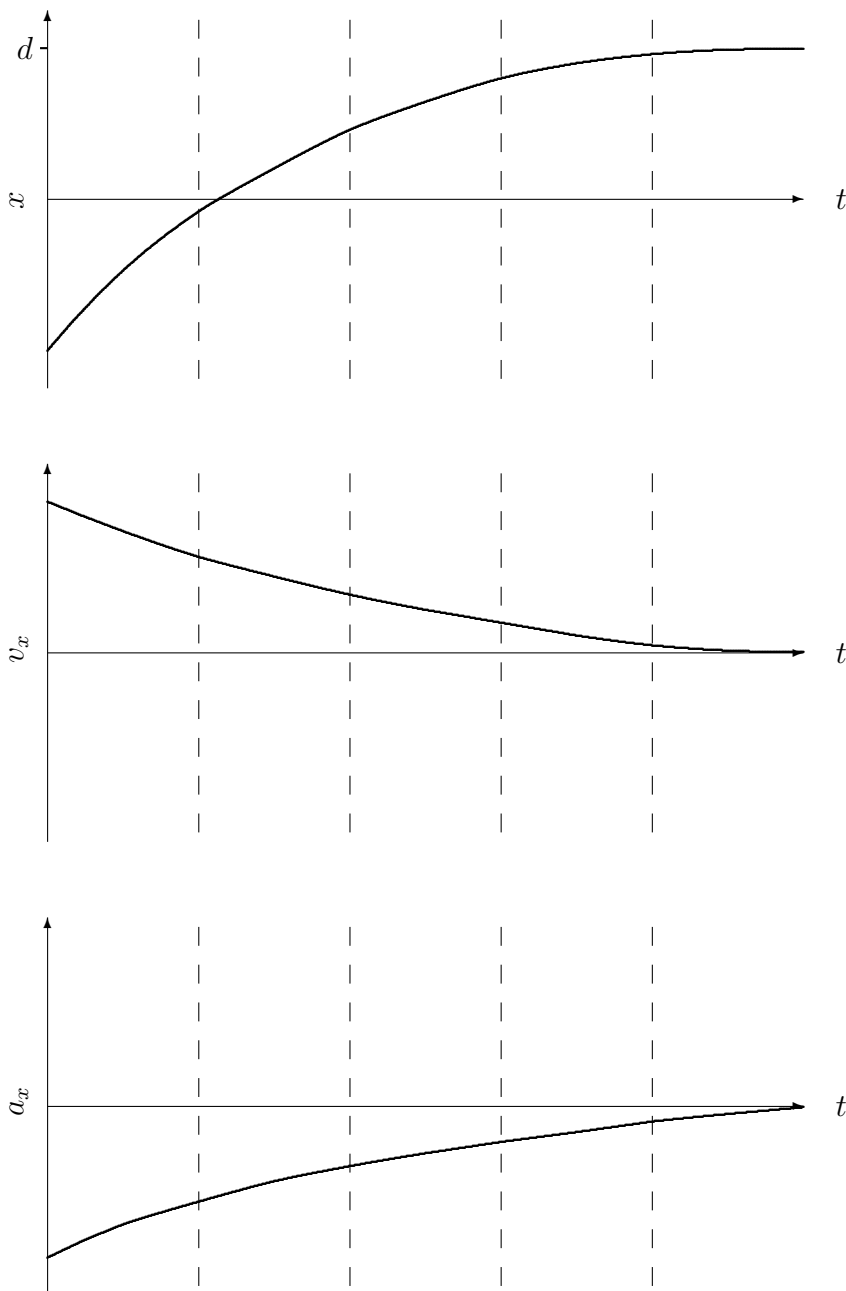


Solutions to Assignment 1; PHYS 185

1. (20 points) The top graph displays how position depends on time for an object that gradually, but ever more slowly, approaches $x = d$. Make a qualitative sketch of the corresponding velocity versus time and acceleration versus time graphs for this motion.

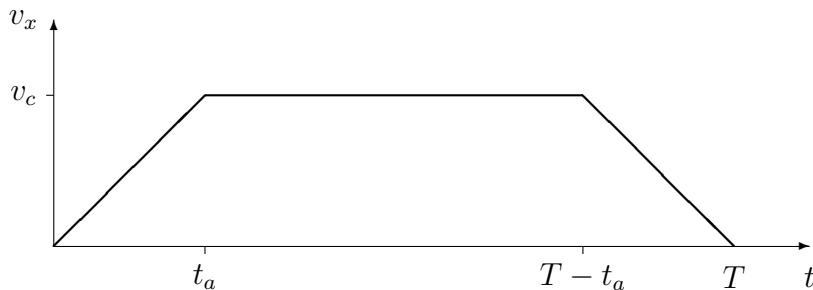


Answer: Notice that the slope of the x - t graph is positive, but that it becomes flatter—the slope gets closer to zero—as t increases. Therefore the velocity graph stays positive,

but decreases.

We look at slopes again to find the acceleration. The velocity is *decreasing*, therefore the acceleration is negative. But since v_x flattens out with time, a_x also approaches zero.

2. (40 points) The distance from London to Sydney is 1.70×10^7 m. You take a supersonic flight that covers this distance in exactly $T = 2.00$ hours. Say the flight takes a time t_a to accelerate from rest, reaching a constant cruising speed of v_c , and then while landing, takes the same time t_a to decelerate. For a commercial flight that will be taken by people with varying health conditions, the magnitude of the horizontal component of the acceleration imposed on the passengers should not exceed $2g$ ($g = 9.80$ m/s²) for longer than three minutes. Calculate t_a for this maximum $a_x = 2g$, and determine whether this flight can be safe.



Answer: The distance covered, let's call it d , is the area under the v - t curve. (You can also do this by treating the three different constant acceleration segments of the motion separately, using the standard motion with constant a_x equations, and adding all of it up, but it amounts to the same thing.)

$$\Delta x = x(T) - x(0) = v_c(T - t_a)$$

Also, from the graph,

$$a_x = \frac{v_c}{t_a}$$

Setting $a_x = 2g$ and combining the two equations, we get a quadratic equation for t_a :

$$d = 2gt_a(T - t_a) \quad \Rightarrow \quad t_a^2 - Tt_a + d/(2g) = 0$$

The solutions to this are, using $d = 1.70 \times 10^7$ m and $T = 7200$ s,

$$t_{a1,2} = \frac{T \pm \sqrt{T^2 - 2d/g}}{2} = 123 \text{ s}, 7077 \text{ s}$$

The first answer, 123 s, is what we need (the other is $T - t_a$). This is a reasonable value, under three minutes, so this should be safe.

3. (40 points) You launch a projectile on a level surface on a planet with acceleration due to gravity g , starting from $x_0 = y_0 = 0$, with initial speed v_0 and angle θ with the x -axis.

But you're facing a strong horizontal wind, so that the motion has a non-zero $a_x = -w$, where w is a positive constant that stands for the magnitude of the acceleration due to the wind.

- (a) Write down the equations for position and velocity components as a function of time:

$$\begin{aligned}v_x(t) &= v_0 \cos \theta - wt & x(t) &= v_0 \cos \theta t - \frac{1}{2}wt^2 \\v_y(t) &= v_0 \sin \theta - gt & y(t) &= v_0 \sin \theta t - \frac{1}{2}gt^2\end{aligned}$$

- (b) Find the *range* of the projectile: an equation for how far it will travel until it hits the ground again.

Answer: You want t for $y = 0$. The wind has no effect on this; you end up with the usual

$$0 = v_0 \sin \theta t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = 0 \text{ or } \frac{2v_0 \sin \theta}{g}$$

where you throw away the $t = 0$ solution. The distance traveled is x at this t :

$$x = v_0 \cos \theta \frac{2v_0 \sin \theta}{g} - \frac{1}{2}w \left(\frac{2v_0 \sin \theta}{g} \right)^2 = \frac{2v_0^2 \sin \theta}{g} \left(\cos \theta - \frac{w}{g} \sin \theta \right)$$

- (c) Check your result: when you set $w = 0$, you should get the same equation for the range as you have in your class notes.

Answer: With $w = 0$, you get

$$x = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

which is what you should have.

- (d) The range is positive when $w < [\text{an expression involving } g \text{ and } \theta]$. Find this inequality. Would it make physical sense for the range to be negative?

Answer: Looking at the equation, you see that $x > 0$ when

$$\left(\cos \theta - \frac{w}{g} \sin \theta \right) > 0 \quad \Rightarrow \quad w < g \cot \theta$$

The wind can be strong enough that the projectile loops backward.

- (e) See what happens when $w = g$ and $\theta = 45^\circ$. Interpret your result in this case—what does the motion look like?

Answer: Notice that since $\sin 45^\circ = \cos 45^\circ$ and $w = g$, the equations for $x(t)$ and $y(t)$ become identical. This means that $x = y$ always, therefore the trajectory of the projectile has to be on the line with a 45° angle with the horizontal—you shoot your projectile straight along that line; it goes diagonally up a bit and comes straight back down.

Extra Problems (not graded)

4. (0 points) You have a cannon that launches rubber balls with an initial speed of $v_0 = 12.6$ m/s. You set it at an angle $\theta = 38^\circ$ above the horizontal, and shoot a ball at a high vertical wall standing a distance $l = 9.20$ m in front of the cannon.

- (a) Find symbolic expressions for v_x and v_y at the instant before the rubber ball hits the wall. Then plug in the numbers and find their values.

Answer: The initial velocity components are $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$. The acceleration components are $a_x = 0$ and $a_y = -g$. We want to find v_x and v_y at the time t when $x = l$. Therefore,

$$l = 0 + v_0 \cos \theta t + 0 \quad \Rightarrow \quad t = \frac{l}{v_0 \cos \theta}$$

At this time,

$$v_x = v_{0x} = v_0 \cos \theta = 9.93 \text{ m/s}$$

and

$$v_y = v_0 \sin \theta - gt = v_0 \sin \theta - \frac{gl}{v_0 \cos \theta} = -1.32 \text{ m/s}$$

- (b) The instant *after* the rubber ball bounces off the wall, the y -component of its velocity remains the same as it was just before it hit the wall. But the x -component of its velocity reverses its direction (same magnitude, opposite sign). Find out where, relative to the cannon, the ball falls back to the ground.

Answer: There are multiple ways to solve this. The easiest is to recognize that reversing v_x means that the motion after the ball hits the wall will be the same as if the wall were not there, but in the $-x$ direction instead. The distance traveled without the wall would be

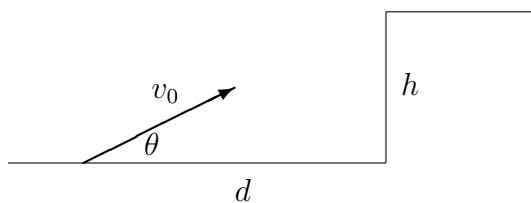
$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

so the ball would have traveled an extra distance of $R - d$. Since the direction is reversed, we *subtract* that from d , find in that ball landed a distance

$$d - (R - d) = 2d - R = 2d - \frac{2v_0^2 \sin \theta \cos \theta}{g} = 2.7 \text{ m}$$

in front of the cannon.

5. (0 points) You're on a moon with no atmosphere, and acceleration due to gravity q . You have a cannon that can shoot a ball out beyond the crater wall with height h a distance d away. The angle of the cannon with the horizontal is θ , and the initial speed of the ball is v_0 .



(a) Find an inequality describing the conditions under which the ball will make it out of the crater.

Answer: No atmosphere: no drag. The equations for free fall should work very well. First start by figuring out when the ball just makes it to the edge of the crater, $x = d$

$$d = 0 + v_0 \cos \theta t \quad \Rightarrow \quad t = \frac{d}{v_0 \cos \theta}$$

At that time,

$$y = 0 + v_0 \sin \theta t - \frac{1}{2} q t^2 \quad \Rightarrow \quad y = d \tan \theta - \frac{q d^2}{2v_0^2 \cos^2 \theta}$$

The condition to make it out is $y \geq h$. Therefore, the inequality we need is

$$d \tan \theta - \frac{q d^2}{2v_0^2 \cos^2 \theta} \geq h$$

(b) Using your inequality, you should find that there is an angle below which the ball will never make it out, no matter how large v_0 is. Find this angle. Then explain what your result means—physically, what is going on?

Answer: When v_0 becomes very large, the second term on the left side of the inequality becomes very small. (For those of you who remember limits: it goes to zero.) Therefore, the inequality becomes

$$d \tan \theta \geq h$$

If, therefore, the angle is such that

$$\tan \theta < \frac{h}{d} \quad \text{or} \quad \theta < \tan^{-1} \frac{h}{d}$$

the ball can never make it out. Notice that this angle describes a triangle that connects the launch point to the lip of the crater by a straight line. At that angle, only a ball launched at infinite speed, which then will have no time to dip down under the influence of gravity, can barely make it out of the crater. Even infinite speed won't do it for smaller angles.