

## College Physics I

# Lab 5: Simple Harmonic Motion

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### Static determination of the spring constant $k$

The magnitude of the force an ideal spring exerts is proportional to its displacement from equilibrium,  $x$ . One way to determine the proportionality constant,  $k$ , for a spring is to hang a mass  $m$  from it. The force that mass is exerting on the spring is then  $mg$ , where  $g$  is the acceleration due to gravity—assume  $g = 9.80 \text{ m/s}^2$ . We can then see how far the spring stretches. More quantitatively:

$$\begin{aligned} |F| &= k|x|, \\ |F| &= mg, \\ \Rightarrow |x| &= \frac{g}{k} m. \end{aligned} \tag{1}$$

Thus if you graph  $|x|$  versus  $m$  (with  $m$  on the horizontal axis), and the data approximate a straight line, that is evidence that the spring is close to ideal. Furthermore, the slope of the line,  $s$ , will be  $g/k$ . You could determine  $k$  if you found  $s$ :  $k = g/s$ .

### Part 1: Determine $k$

Find  $k$ . Use 10 different masses, but don't add more than 500  $g$  total, as more than that may permanently deform the spring.

#### To hand in for part 1

- Values of  $m$  and  $|x|$  for each trial,

- Graph of  $|x|$  versus  $m$ ,
- Calculation of  $s$  and the final result for  $k$ .

## When the spring is moving

The larger the spring constant  $k$ , the “stiffer” the spring— $k$  is often called the *stiffness* of the spring. As it happens,  $T$ , the period of vibration of a mass  $m$  bouncing on a spring is

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (2)$$

In this equation,  $k$  should be constant. Therefore  $T$  is equal to a constant times  $\sqrt{m}$ , or equivalently,  $T^2$  is proportional to  $m$ . So if you put a bunch of different masses on a spring and measure the period for each mass, and then graph  $T^2$  versus  $m$ , you should get a straight line with a slope of  $(2\pi)^2/k$ :

$$T^2 = \left[ \frac{(2\pi)^2}{k} \right] m \quad (3)$$

The most precise way to measure the period is to measure, say, 50 periods, and then divide by 50. That way, any imprecision in your measuring process is spread out over a much greater time, and will have a much smaller overall effect. If  $n$  is the number of periods you time, and  $t_n$  is the time for all those periods, then:

$$T = \frac{t_n}{n} \quad (4)$$

You will have to face an extra complication in this part of the lab. I want you to figure out a way of measuring time *without* using any watches, clocks, or other device that is designed to tell time. This is, in fact, a problem the first experimental physicists faced, before devices to accurately measure short periods of time became available.

## Part 2: Period as a function of mass

Think of a method of measuring time intervals. Call your basic time interval  $\tau$ , so that you will be measuring time in units of  $\tau$ 's rather than in seconds.

You will, for example, say that each oscillation takes 3.4  $\tau$ 's. Note that as long as  $\tau$  is comparable to the period of the oscillations and fairly constant, you will do fine. Just to check whether  $T^2$  is proportional to  $m$  does not require that time be measured in any particular units, so don't worry if you can't tell exactly how many seconds  $\tau$  happens to be.

Now, using your method of timing, find  $T$  for ten different values of hanging mass ranging from 50 g up to about 300 g. Then make a graph of  $T^2$  versus amount of hanging mass, again with  $m$  on the horizontal axis. With an ideal spring, you should get a straight line.

In fact, if all went according to expectations, calculating the slope of the line in this graph should give you what  $\tau$  is, in seconds. Equation (3) means that your slope will be  $(2\pi)^2/k$ , but where the time units are  $\tau$ 's rather than seconds. Use the  $k$  you have already found to calculate the slope in units of seconds, and do the unit conversion to get how many seconds  $\tau$  is.

You can then ask me for a stopwatch to measure  $\tau$  directly. Keep in mind that due to human reaction time, a single measurement will not be very accurate.

## To hand in for part 2

- A description of your method of timing.
- Values of  $m$ ,  $t_n$ ,  $n$ ,  $T$  and  $T^2$  for each trial (time measured in  $\tau$ 's).
- Graph of  $T^2$  versus  $m$ , and the slope.
- The expression relating slope and  $\tau$  in seconds.
- Your measurement for  $\tau$  using a stopwatch.