Solutions to Assignment 2; PHYS 185

1. (30 points) You're in a spaceship traveling to a nearby star, in deep space far from large gravitational influences.

(a) Is there any way you can tell the difference between moving at an extremely high constant speed toward your destination, and idling at rest going nowhere, *without receiving information from outside the spaceship*? Explain why.

Answer: There is no way. Since force is proportional to acceleration, all forces inside the spaceship will look exactly the same if the acceleration is the same. And in both cases $\vec{a} = 0$ since $\vec{v}$ is not changing.

(b) Say you're close to your destination, and slowing down at a constant acceleration. You need to slow down from 1.00×10^7 m/s (about 3% of light speed) to 0 in 10.0 days. Your mass is 60.0 kg. As you stand on a surface perpendicular to the direction of your acceleration vector, what is magnitude of the normal force you will feel? Compare this to the normal force you would feel standing on the ground back on Earth. Is it safe?

Answer: The acceleration involved is

$$a = \frac{\Delta v}{\Delta t} = \frac{10^7 \text{ m/s}}{60 \times 60 \times 24 \times 10 \text{ s}} = 11.6 \text{ m/s}^2$$

This will be provided by the normal force, since there will be no other forces involved. Therefore $n = ma = 696$ N, compared to $n = mg = 588$ N on Earth. This is 18% higher; uncomfortable but bearable for ten days.

2. (30 points) In Lab 3, "Acceleration due to Gravity," you made a number of simplifying assumptions. In the list below, describe how neglecting each effect made you slightly underestimate or overestimate g as calculated from your data. (I include the answer for the first as an example.) Also draw and label arrows for *every* force you know of on the diagram, including those that you neglected as being too small to worry about, and including forces on the string and pulley.

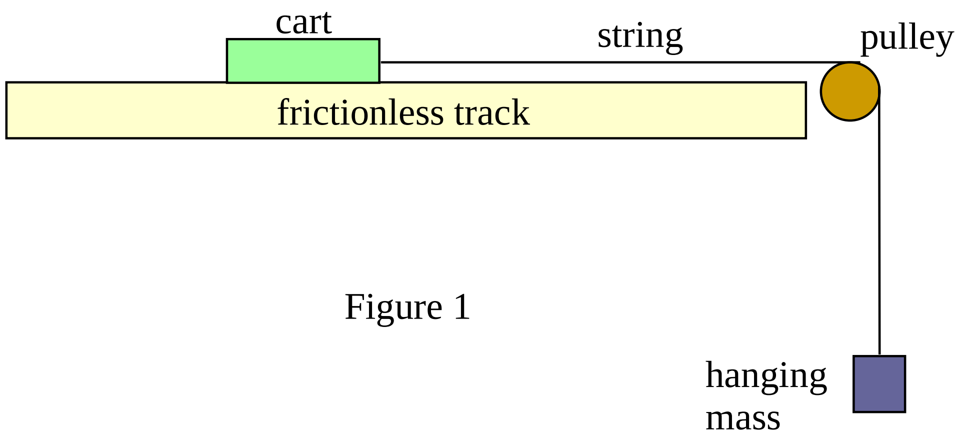

Figure 1

(a) **Track not frictionless:** A small extra kinetic friction force on the cart toward the left slows the cart down, increasing the measured Δt. The equation for g was

$$g = \frac{2(m_{\text{hanging}} + m_{\text{cart}})\Delta x}{m_{\text{hanging}}\,(\Delta t)^2}$$

With increased Δt, friction will cause us to underestimate g.

(b) **Drag on the cart:** Though not constant, drag, like friction, opposes the motion. Its effect will be same, leading to an underestimate of g.

(c) **Track not exactly level; cart goes slightly downhill:** A downhill tilt will add a component of the weight in the forward direction, making the cart go faster and reducing the measured Δt. So the effect is the opposite of friction: it will lead to an overestimate of g.

(d) **Drag on hanging mass:** This drag will be in the same direction as the tension in the string, therefore reducing the tension opposing the weight of the hanging mass. The same tension pulls the cart, which will now be smaller. This increases Δt and leads to an underestimate of g.

(e) **Initial speed of cart into first photogate not quite zero:** Being faster will reduce Δt and lead to an overestimate of g.

(f) **Pulley not frictionless:** Friction in the pulley will also increase Δt and lead to an underestimate of g.

(g) **Pulley not massless:** This means that a force is needed to accelerate the rotation of the pulley. Imagine what would happen if the pulley mass was really large: everything would slow down to a crawl. Therefore, a massive pulley increases Δt and leads to an underestimate of g.

3. (40 points) You have a book sliding on a table, and the coefficient of kinetic friction between the book and the table is μ_k. The table makes an angle of ϕ with the *vertical* (not the horizontal!). ϕ is small enough that if you placed the book at rest on the table, it would slide down. You now launch the book up the slope of the table with an initial speed v_0.

(a) What distance along the table will the book travel before it starts sliding back down?

Answer: Using the usual tilted coordinate axes,

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi - \mu_k n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi + \mu_k \sin\phi)$$

Now we do motion with constant acceleration. To come to a rest,

$$v_x = v_0 + a_x t = 0 \qquad \Rightarrow \qquad t = \frac{v_0}{g(\cos\phi + \mu_k\sin\phi)}$$

Choosing to call the point where the book starts $x_0 = 0$,

$$x = v_0 t + \frac{1}{2}a_x t^2 = \frac{v_0^2}{2g(\cos\phi + \mu_k\sin\phi)}$$

(b) The book now slides down. Find its velocity as it passes the point it was launched from. (In other words, the book slides down the same distance it traveled up in part a).

Answer: On the way down, the friction force reverses direction, but otherwise everything remains the same.

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi + \mu_k n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi - \mu_k \sin\phi)$$

Using x from part (a) as the new x_0, and looking to see what happens when the book goes through $x = 0$ again, we get

$$0 = \frac{v_0^2}{2g(\cos\phi + \mu_k\sin\phi)} + \frac{1}{2}a_x t^2 \quad \Rightarrow \quad t = \frac{v_0}{g\sqrt{(\cos\phi + \mu_k\sin\phi)(\cos\phi - \mu_k\sin\phi)}}$$

$$v_x = a_x t = -v_0\sqrt{\frac{\cos\phi - \mu_k\sin\phi}{\cos\phi + \mu_k\sin\phi}}$$

Extra Problems (not graded)

4. (0 points) Say you're down in the introductory physics lab, and you have all the equipment you have used so far available to you: low-friction track, motion detector, masses and mass hangers, string, rulers, tape, scissors, card stock, boxes, photogates, electronic scales, stopwatches, small pulley wheels, etc. etc.

Now say I also give you a wooden block about the same size and shape as a typical textbook. The block also has a couple of hooks on it in case you want to attach anything. And now I tell you that your job is to measure μ_s, the coefficient of static friction between the block and the stone surface of the lab table.

Design an experiment that will allow you to obtain μ_s. Be sure to include:

- A clear description of your setup and procedure.
- A list of the equipment you need.
- A clearly labeled diagram of your setup, with arrows indicating the relevant forces.
- If you are going to obtain μ_s by using an equation, a derivation of the equation and a clear indication of how you measure the various quantities that go into it. (Standard values such as g, you can just look up.)
- If you're going to use a graph rather than an equation, a drawing of an example graph, and a description of how you would use that graph to obtain μ_s.

Hint: Keep it simple! There are lots of ways you can measure μ_s, but some of the best ways are also reasonably simple. So don't trip yourself up trying to be overly elaborate.

Answer: There are many ways to do this. The most straightforward is probably to tie a string to the block and let it go over a pulley, with masses attached to the end. You add hanging masses until you reach m_h, when the book just starts sliding across the table. In that case, the balance of forces means $m_h g = \mu_s n = \mu_s m_b g$, where m_b is the mass of the block. Your equation would then be $\mu_s = m_h/m_b$.

In that case, m_b would be obtained by the electronic scale, and m_h by adding up the hanging masses.

5. (0 points) As in your question 3, you slide a book up a table. The coefficient of kinetic friction between the book and the table is μ_k, and the coefficient of static friction is μ_s. The table makes an angle of ϕ with the *vertical* (not the horizontal!). You launch the book up the slope of the table with an initial speed v_0. Is it possible that the book will get stuck at its topmost point, and not slide down again? If it is impossible, demonstrate why. If it is possible, find the conditions under which the book will get stuck.

Answer: If the angle ϕ is large enough, static friction at the point where the book would turn around (and where $v_x = 0$) can be strong enough to prevent the book from sliding down.

So, the force components will be just as in question 3, but with kinetic friction replaced by static friction. For the minimum angle ϕ, set f_s to its maximum value, $f_s = \mu_s n$. We then have, using the usual tilted coordinate axes,

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi + \mu_s n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi - \mu_s\sin\phi)$$

But since the book remains at rest, $a_x = 0$. Therefore, canceling out g,

$$\mu_s = \frac{\cos\phi}{\sin\phi}$$

That was the lower limit. The condition we're looking for is therefore

$$\phi \geq \cot^{-1}\mu_s$$