Solutions to Assignment 3; PHYS 185

1. (20 points) You have an object with mass m moving on a flat surface, released with initial velocity v_i. The coefficient of kinetic friction between the surface and the mass is μ_k.

(a) Find the distance the object will travel before coming to a halt, in terms of v_i, μ_k, m, and g.

Answer: Adding up the forces will give the acceleration components:

$$\sum F_y = n - mg = ma_y = 0 \qquad \Rightarrow \qquad n = mg$$

$$\sum F_x = -f_k = -\mu_k n = -\mu_k mg = ma_x \qquad \Rightarrow \qquad a_x = -\mu_k g$$

Notice that the mass cancels out. With constant acceleration, we have $v_f = 0$ and

$$0 = v_i - \mu_k gt \quad \Rightarrow \quad t = \frac{v_i}{\mu_k g}, \qquad x - x_0 = v_i t - \tfrac{1}{2}\mu_k g t^2 = \frac{v_i^2}{2\mu_k g}$$

(b) You also have a second object which is released with the same initial velocity v_i. This second object is placed in a sleeve that reduces friction by 1%, so that its coefficient of kinetic friction with the surface is $0.99\mu_k$, but increases its total mass by 1%, so that its mass is $1.01m$. Which object will travel a larger distance before coming to a halt?

Answer: In the calculation above, the mass cancels out, so an increased mass is irrelevant. So, with less friction, the second object will travel farther.

A calculation produces the same result, only replacing μ_k with $0.99\mu_k$. Therefore

$$a_x = -0.99\mu_k g \qquad \text{and} \qquad \Delta x = \frac{v_i^2}{1.98\mu_k g}$$

This Δx is smaller than the result in (a); the second object travels a larger distance.

2. (40 points) You drop a spherical 4.1 kg object from a bridge with height 84 m over the water. Take $C_D = 0.5$, the density of air $\rho = 1.22$ kg/m^3, and the radius of the object to be 0.25 m.

(a) Assuming you can ignore air resistance, calculate the speed at which the sphere will hit the water, and the time it will take to reach the water.

Answer: Free fall starting from rest: $0 = y_0 - \frac{1}{2}gt^2$, therefore the time it takes to reach the water is $t = \sqrt{2y_0/g} = 4.1\,\text{s}$. The velocity will be $v_y = -gt = -41\,\text{m/s}$, for a speed of 41 m/s.

(b) But we really can't ignore the air resistance. Calculating the motion by accounting for drag force requires calculus, so we won't do that. Instead, circle the most accurate *approximate model* from amongst the following options:

i. Assume constant acceleration of g until the sphere reaches terminal velocity, and afterwards have it continue falling with this constant terminal velocity.

ii. Assume a constant velocity equal to the terminal velocity downward throughout the fall.

iii. Assume constant acceleration throughout, but reduce the downward acceleration to $g - \frac{1}{2}C_D m$, where m is the mass, to account for the drag force.

iv. Assume constant acceleration throughout, but reduce the acceleration to $g\frac{1}{2}C_D$, to account for the effect of the drag force.

Explain why you think your choice is the best approximation among these options.

Answer: Option **(i)** is best. First, eliminate **(iii)** and **(iv)**. Option **(iii)** attempts to subtract mass from acceleration and is therefore nonsense. Option **(iv)** has an equation that has the correct units, but is still strange. Since the acceleration a constant, there will be no terminal velocity. And the acceleration magnitude will increase as C_D increases, which means that the more drag, the larger the acceleration. That doesn't make sense.

Among the remaining two options, the first does a better job accounting for the fact that the object has to speed up to achieve terminal velocity.

(c) Use your choice of approximate model to calculate the speed at which the sphere will hit the water, and the time it will take to reach the water. Compare these to your answers from (a)—do they make sense?

Answer: The terminal velocity is achieved when $\frac{1}{2}C_D\rho A v^2 = mg$, therefore $v_T = \sqrt{2mg/C_D\rho A} = 26$ m/s. (A, here, is the cross-sectional area of the sphere, πr^2.) If we accelerate at g to reach this speed, we need, from $v_T = gt$, $t = v_T/g = 2.6$ s. During this time, the object will have fallen $\frac{1}{2}gt^2 = 34$ m.

The final $84 - 34 = 50$ m will be at constant velocity v_T. This takes $50/26 = 1.9\,\text{s}$.

So in this model, the whole fall takes $2.6 + 1.9 = 4.5\,\text{s}$, and the sphere hits the water at a speed of $v_T = 26\,\text{m/s}$. This is $|4.5 - 4.1|/4.1 = 10\%$ longer, and $|26 - 41|/41 = 37\%$ slower than (a).

3. (40 points) Lab 3 was not meant to measure g accurately. But you decide to fix some of your sources of error and do a better job. Fist, you devise an automatic release mechanism for the cart, so that it does in fact start with a zero initial velocity as it heads into the first photogate. Then you make some calculations, establishing that the cart never goes fast enough to take the drag force into consideration, and that the pulley's mass and friction at the pulley are so small that they shouldn't matter. And so you set up the equipment and measure Δt and the distance between the photogates, calculating g by using the equation from Lab 3.

(a) Your g result turns out to be improved, but still not good. You then realize that you didn't account for the track possibly being tilted, and that there was friction between the cart and the track. No matter: you realize that if you could measure the tilt angle θ and the coefficient of kinetic friction μ_k, you could still use your data from the experiment, as long as you altered the equation for g to account for the tilt and the friction. Find this improved equation for g.

Answer: With tilted coordinate axes and friction, the force components on the cart are

$$w_x = +m_c g \sin\theta \quad w_y = -m_c g \cos\theta \qquad n_x = 0 \quad n_y = +n$$
$$T_x = +T \quad T_y = 0 \qquad f_{kx} = -\mu_k n \quad f_{ky} = 0$$

where m_c is the cart mass. With m_h, the hanging mass, we get the forces on the . The tension force is still $T = m_h g$. The normal force makes sure $a_y = 0$, so

$$\sum F_y = n - m_c g \cos\theta = 0 \qquad \Rightarrow \qquad n = m_c g \cos\theta$$

$$\sum F_x = m_c g \sin\theta + T - \mu_k n = m_c a_{cx} \qquad \Rightarrow \qquad a_{cx} = (\sin\theta - \mu_k \cos\theta)\, g + \frac{T}{m_c}$$

With m_h the hanging mass, the forces on the hanging mass give

$$\sum F_y = T - m_h g = m_h a_{hy} \qquad \Rightarrow \qquad T = m_h(g + a_{hy})$$

Now, since the cart and the hanging mass are connected, $a_{cx} = -a_{hy} = a$, where a is the theoretical acceleration in Lab 3. Doing the algebra and solving for a, we get

$$a = (\sin\theta - \mu_k \cos\theta)\, g + \frac{m_h(g-a)}{m_c} \qquad \Rightarrow \qquad a = \frac{\left(\sin\theta - \mu_k\cos\theta + \frac{m_h}{m_c}\right)}{1 + \frac{m_h}{m_c}}\, g$$

A form closer to the lab equation is

$$a = \frac{(\sin\theta - \mu_k\cos\theta)\, m_c + m_h}{m_h + m_c}\, g$$

Setting a equal to the experimental acceleration of $2\Delta x/(\Delta t)^2$, we get

$$\frac{(\sin\theta - \mu_k\cos\theta)\, m_c + m_h}{m_h + m_c}\, g = \frac{2\Delta x}{(\Delta t)^2} \qquad \Rightarrow \qquad g = \frac{2\Delta x}{(\Delta t)^2}\frac{(m_h + m_c)}{(\sin\theta - \mu_k\cos\theta)\, m_c + m_h}$$

(b) Check that your answer is the same as what is given in the pre-lab for Lab 3 when $\theta = 0$ and $\mu_k = 0$.

Answer: Since $\sin 0 = 0$ and $\cos 0 = 1$, we end up with

$$g = \frac{2\Delta x}{(\Delta t)^2} \frac{(m_h + m_c)}{m_h}$$

That's what you used in the lab.

(c) Propose an experiment you can perform in the lab with equipment that you've used so far, that will allow you to determine μ_k for the friction between the cart and track. Keep your experiment simple; for example, don't incline the track—that's an unnecessary complication.

Answer: Here is an easy option. Set up a flat track, and release the cart on it, using a motion detector to measure its acceleration. If you send the cart toward the motion detector, the friction and the acceleration will be in the $+x$ direction, so

$$\mu_k n = \mu_k m_c g = m_c a_x \qquad \Rightarrow \qquad \mu_k = \frac{a_x}{g}$$

Note that you should not re-use the set-up for Lab 3, since you want an *independent* handle on g. It would be possible to treat g as an unknown here and put it back into the part (a) equation and solve for g again, but you could not do that if you used your result from (a) to get μ_k in the first place.

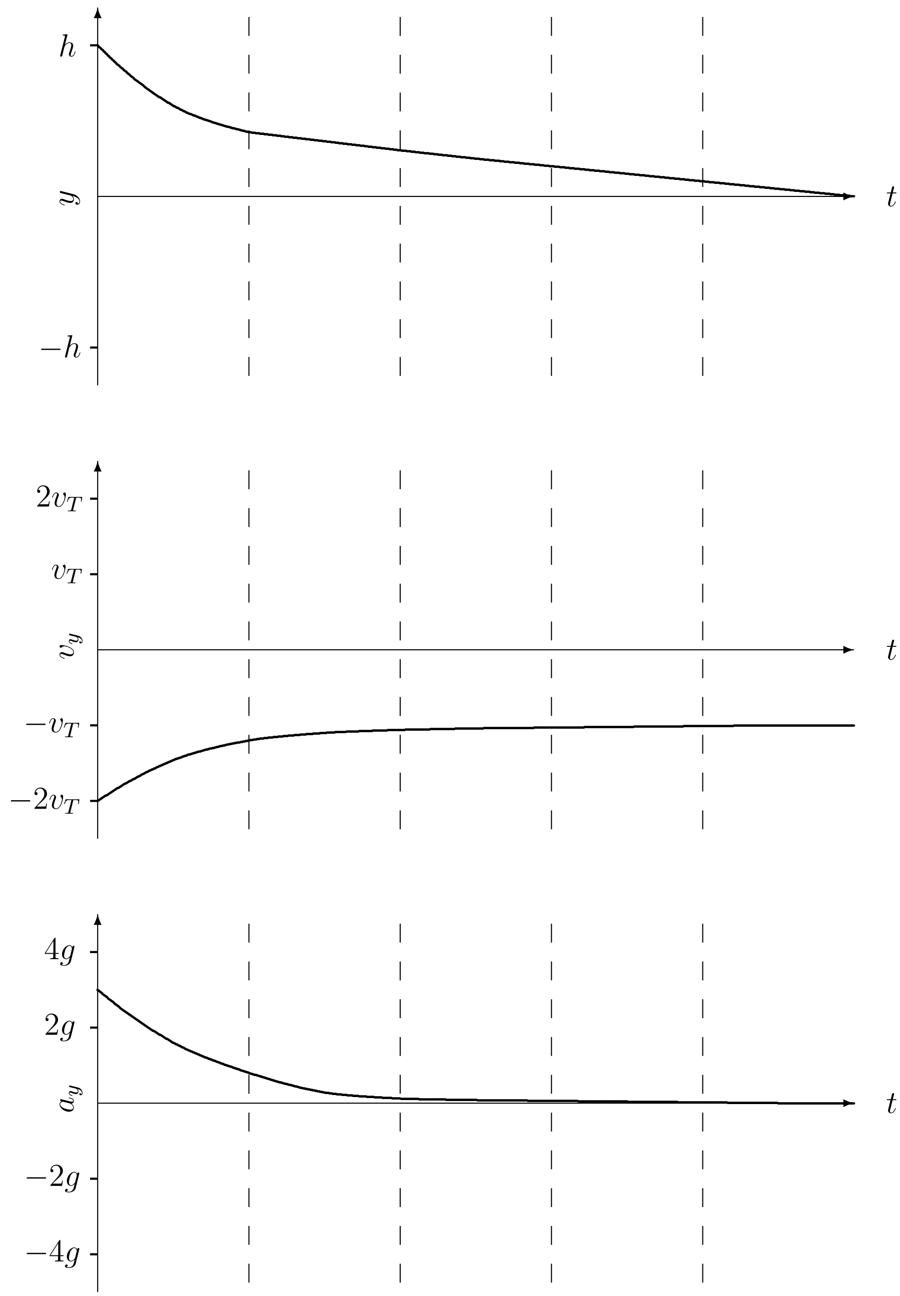
Extra Problems (not graded)

4. (0 points) You go to the top of a very high skyscraper with height h, and shoot a ball directly downwards. Air resistance, however, is not negligible, and the initial speed of your ball as it heads down is $2v_T$, where v_T is the terminal speed for the ball. Make *qualitative* sketches of the ball's $y(t)$, $v_y(t)$, and $a_y(t)$. *Hint:* You can figure out exact values for all three at $t = 0$; start there.

Answer: First, $t = 0$. We have $y(0) = h$ and $v_y(0) = -2v_T$ given. (Note the $-$ to indicate the direction.) The acceleration needs more thought. Since v_T is the speed at which the drag force cancels out the weight, when $v = v_T$, $D = mg$. But what about $2v_T$? The drag force depends on the *square* of the speed: $D \propto v^2$. If you double the speed, you quadruple the drag. Therefore, at $t = 0$, the drag must be $D = 4mg$. The drag opposes the velocity, so it points upward. In other words, the force components are $D_y = 4mg$ and $w_y = -mg$. Add the forces:

$$\sum F_y = 4mg - mg = 3mg = ma_y \qquad \Rightarrow \qquad a_y(0) = 3g$$

Therefore v_y will increase (become less negative) as the ball falls, which means v^2 becomes *smaller*. So D will decrease, $\sum F_y$ will decrease, and so will a_y. As v_y rises toward $-v_T$, it will therefore change less rapidly, gradually approaching its terminal value $-v_T$, where $\sum F_y = 0$ and $a_y = 0$.



5. (0 points) You're doing the experiment in Lab 2, with the cart going up and down an inclined low-friction track. You notice that the accelerations going up and down are slightly different; $a_{\text{up}} = -2.1\ \text{m/s}^2$ and $a_{\text{down}} = -1.9\ \text{m/s}^2$; where these are accelerations along an x-axis tilted to be parallel with the track, with the $+x$-direction pointing up (away from the motion detector).

(a) Find θ, the angle at which the track is tilted. *Hint:* If you solve this symbolically, you'll find that θ depends on $\frac{1}{2}(a_{\text{up}} + a_{\text{down}})$ and g.

Answer: The only difference between the up and down motions is that the kinetic friction force reverses direction, since it points opposite the velocity. The weight and normal forces remain the same. Going up,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta - \mu_k mg\cos\theta = ma_{\text{up}} \qquad \Rightarrow \qquad a_{\text{up}} = -g(\sin\theta + \mu_k\cos\theta)$$

Coming down,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta + \mu_k mg\cos\theta = ma_{\text{down}} \qquad \Rightarrow \qquad a_{\text{down}} = -g(\sin\theta - \mu_k\cos\theta)$$

If we add the equations for a_{up} and a_{down}, we get

$$a_{\text{up}} + a_{\text{down}} = -2g\sin\theta \qquad \Rightarrow \qquad \theta = \sin^{-1}\left[-\left(\frac{a_{\text{up}} + a_{\text{down}}}{2}\right)\frac{1}{g}\right] = 12^\circ$$

(b) Find μ_k, the coefficient of kinetic friction between the track and the cart. *Hint:* If you solve this symbolically, you'll find that μ_k depends on $\frac{1}{2}(a_{\text{up}} - a_{\text{down}})$, g, and θ.

Answer: If we now subtract the equations for a_{up} and a_{down}, we get

$$a_{\text{up}} - a_{\text{down}} = -2\mu_k g\cos\theta \qquad \Rightarrow \qquad \mu_k = -\left(\frac{a_{\text{up}} - a_{\text{down}}}{2}\right)\frac{1}{g\cos\theta} = 0.010$$

This is quite small, as befits a low friction track.