Solutions to Assignment 5; Phys 185

1. (30 points) Why are brakes on a bicycle applied to the rim of a wheel and not on the axle? Model the bicycle wheel as a disk. Apply the same normal force with the brake pad, with the brake pad placed either near the rim or near the axle. The materials that come into contact are the same in both cases. In which case would the wheel come to rest faster? Construct a full argument, using equations as appropriate.

Answer: Stopping faster requires a higher rate of change of angular velocity: a large angular acceleration α. The disk's moment of inertia I does not depend on where the brake is applied. Therefore $\sum \tau = I\alpha$ means that the better location of the brake is that which produces a higher torque τ, which will result in a higher α slowing down. The force producing τ is kinetic friction $f_k = \mu_k n$. n is the same for both cases, and since the materials that come into contact are the same, then so is μ_k. Friction is perpendicular to the line between the point of application and the axis of rotation in both cases. So the only difference is the distance of the brake application point to the axis of the wheel. The larger torque will be produced by the brake that is farther away, since $\tau = f_k r$. Therefore the brake on the rim—with the larger r—will stop the wheel faster.

For future reference: you can also use energy, once we learn about that. Stopping the wheel means converting all its rotational kinetic energy $\frac{1}{2}I\omega^2$ into ΔE_{th}. This thermal energy comes from the work done by friction at the brakes; $\Delta E_{th} = f_k \Delta x$, where Δx is the distance on the wheel over which friction acts. This distance is $\Delta x = r\Delta\theta$, with the change in angle in radians. You need the same Δx in either case, as the kinetic energy to dissipate is the same. The faster stop happens in the case where you have the smaller $\Delta\theta$ before you stop—that corresponds to the larger α. This means that you have to have a larger r—the rim.

2. (30 points) You have a car traveling down the highway at constant speed v. Its wheels have a radius of 0.29 m and have 12 identical-looking spokes each, and they are rolling without slipping. You then film this car at a frame rate of 24 frames per second. Calculate the minimum speed v at which the tires will appear not to be rotating at all on film—so that at slightly smaller speeds the car wheels will appear to be rotating in the wrong direction. Express this speed both in m/s and in miles per hour.

Answer: If there are n spokes, we want the angle $\Delta\theta = 2\pi/n$ covered in time $\Delta t = 1/f$, where f is the frame rate. Then the angular velocity of the wheel is

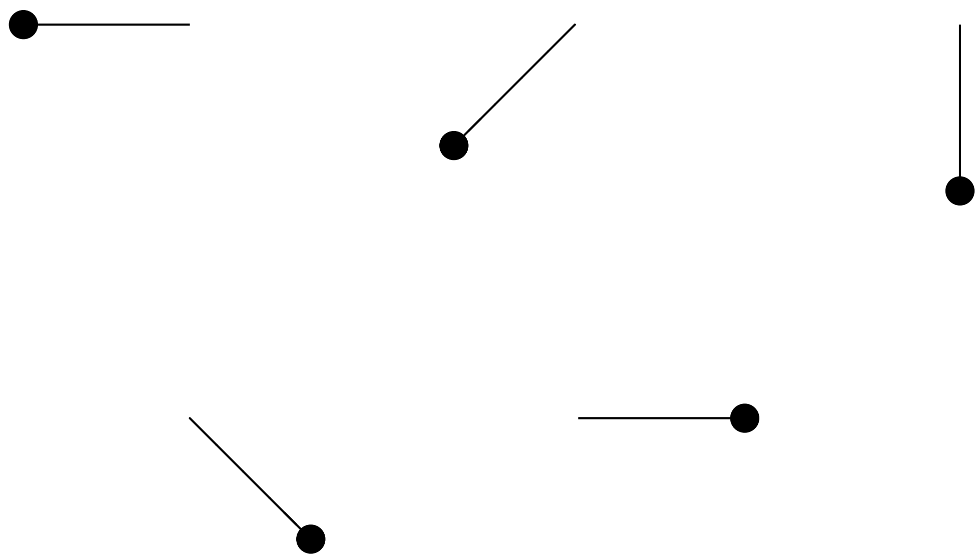
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi f}{n}$$

With rolling without slipping,

$$v = \omega r = \frac{2\pi f r}{n} = 3.64 \text{ m/s} = 8.15 \text{ mph}$$

3. (40 points) You have a pendulum consisting of a mass m hanging at the end of a string with length l. You attach the loose end of the string to the ceiling. Then you start with the string stretched out horizontally, making an angle of $\theta = 0$ with the horizontal. You let the pendulum swing, and since there is no friction or air resistance, the pendulum will swing back and forth with period T, ending up at its initial position after a time T elapses.

(a) Find the torque on the swinging mass, as a function of $m, g, l,$ and θ. Give torque values for the angles shown: $\theta = 0°, 45°, 90°, 135°, 180°$.



Answer: The tension in the string does not contribute to torque, since it is always directed toward the axis of rotation. So the total torque is due entirely to the weight: $\tau = mgl \cos\theta$.

At $\theta = 0$, the weight is perpendicular to the line between the weight's point of application and the axis of rotation. Therefore $\tau = mgl$, with a plus sign because it is counterclockwise.

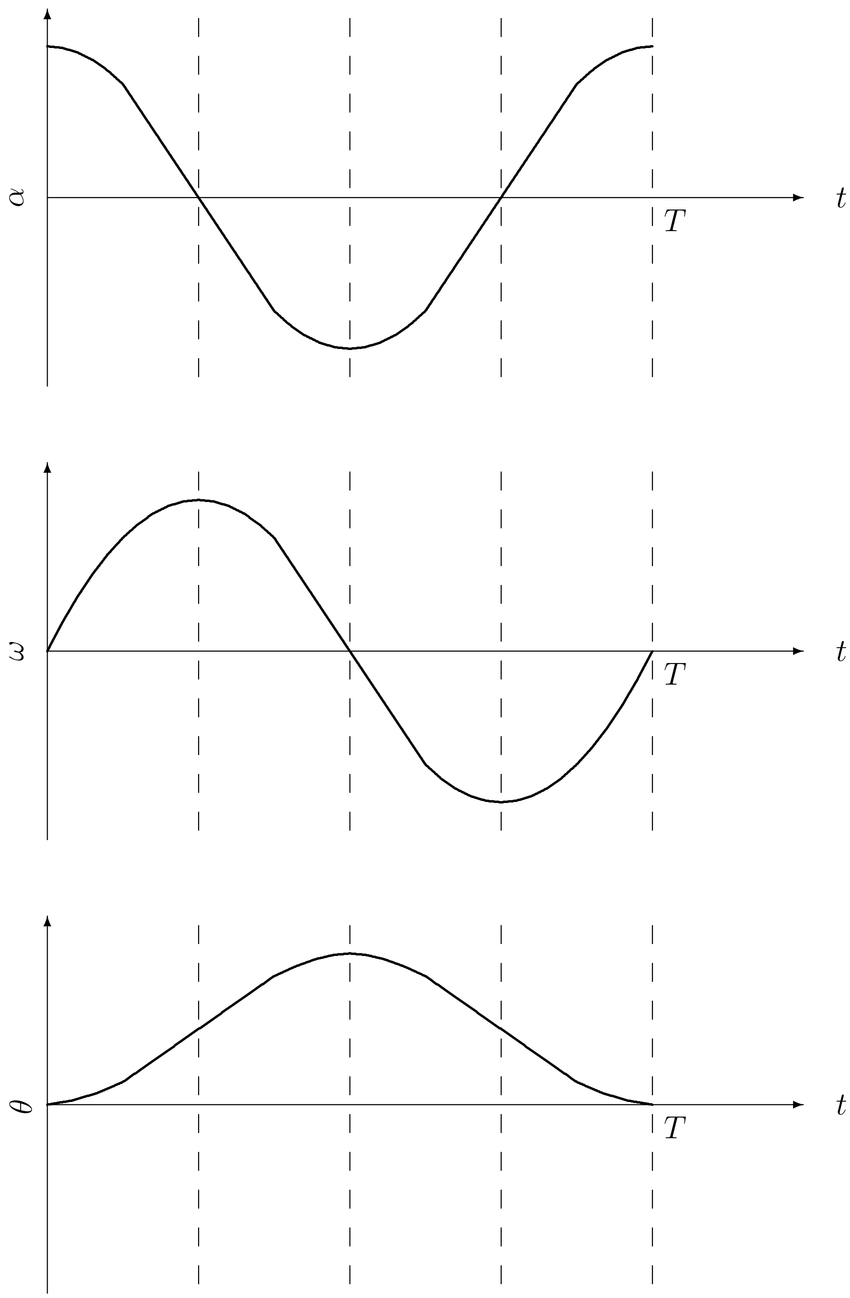
At $\theta = 45°$, the perpendicular component of the weight that contributes to the torque is $mg \cos 45° = mg/\sqrt{2}$. Therefore the torque is $\tau = mgl/\sqrt{2}$.

At $\theta = 90°$, the weight produces no torque, $\tau = 0$.

At $\theta = 135°$, the torque is like that for 45°, except that it is clockwise, so $\tau = -mgl/\sqrt{2}$.

And $\theta = 135°$ is just the opposite of $\theta = 0$, so $\tau = -mgl$.

(b) Make rough sketches of the angular acceleration, angular velocity, and angle versus time graphs. You can't find these exactly without solving some nasty differential equations, but you can produce qualitative graphs. Pay particular attention to the times $0, T/4, T/2, 3T/4, T$; indicating whether any of your quantities are 0, a maximum, or a minimum value at those times.



(These are qualitative sketches, not exact results.)

Extra Problems (not graded)

4. (0 points) You will need to look up astronomical data for the masses, average orbital radii, and rotational periods of the Sun and various planets for the following questions. For example, the Sun's period of rotation around its own axis is 24.5 days.

(a) About where is the axis of rotation of our solar system located? Is this exact, or an approximation? Explain.

Answer: The mass of the Sun is much larger than all the other planets: over a thousand Jupiters. They all rotate about their common center of mass, but the center of mass is almost at the center of the Sun.

(b) Calculate the moments of inertia of the Sun, Earth, and Jupiter. Be explicit about what approximations you are using to get the moment of inertia for each.

Answer: For the sun, use the moment of inertia for a sphere spinning on an axis through its own center; $I = \frac{2}{5}MR^2$. For all the planets, you will notice from the astronomical data that their distances to the Sun are all much larger than their radii. Therefore the point particle approximation will work for them.

$$I_S = \frac{2}{5}(1.99 \times 10^{30} \text{ kg})(6.96 \times 10^8 \text{ m})^2 = 3.86 \times 10^{47} \text{ kg} \cdot \text{m}^2$$

$$I_E = (5.98 \times 10^{24} \text{ kg})(1.50 \times 10^{11} \text{ m})^2 = 1.35 \times 10^{47} \text{ kg} \cdot \text{m}^2$$

$$I_J = (1.90 \times 10^{27} \text{ kg})(7.78 \times 10^{11} \text{ m})^2 = 1.15 \times 10^{51} \text{ kg} \cdot \text{m}^2$$

(c) Calculate the angular momentum of the Sun, Earth, and Jupiter. Add all these angular momenta together to get a total, and state what percentage of this total is associated with each.

Answer: Use $\omega = 2\pi/T$, where T is the period, and $L = I\omega$. 1 year is 3.15×10^7 seconds.

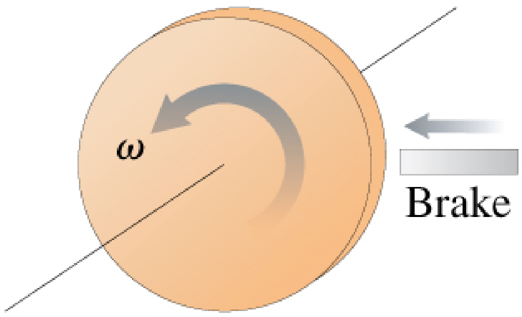
$$L_S = (3.86 \times 10^{47} \text{ kg} \cdot \text{m}^2)\frac{2\pi}{(2.12 \times 10^6 \text{ s})} = 1.14 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$L_E = (1.35 \times 10^{47} \text{ kg} \cdot \text{m}^2)\frac{2\pi}{(3.15 \times 10^7 \text{ s})} = 2.68 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$L_J = (1.15 \times 10^{51} \text{ kg} \cdot \text{m}^2)\frac{2\pi}{(11.9)(3.15 \times 10^7 \text{ s})} = 1.93 \times 10^{43} \text{ kg} \cdot \text{m}^2/\text{s}$$

The total is $L = 2.05 \times 10^{43}$ kg $\cdot$ m^2/s. 5.7% of this belongs to the Sun, 0.13% to Earth, 94% to Jupiter. Most of the angular momentum of the solar system is in the rotation of Jupiter.

5. (0 points) The 2.5 kg, 0.16 m radius disk in the figure is spinning at 28 rad/s. If the brake applies a friction force of 4.2 N to the rim of the disk, how long will it take for the disk to come to a halt?



Answer: Use $\sum \tau = I\alpha$. Any forces on the axle will produce no torque, since their distance to the axis of rotation is zero. The normal force at the brake contact is completely parallel to the line connecting it to the axis of rotation; it has zero perpendicular component. The friction, on the other hand, is completely perpendicular to that line, and it will be the only force producing a torque.

Look up the moment of inertia for a disk: $I = \frac{1}{2}mr^2$. Since the angular acceleration is a constant, $\alpha = \Delta\omega/\Delta t$. Putting it all together,

$$f_k r = \frac{1}{2}mr^2\frac{\Delta\omega}{\Delta t}$$

Solving, we get $\Delta t = 1.33$ s.