Assignment 7; Phys 185 Name ________________

1. (20 points) You're in a spaceship in a circular orbit around a planet. Your distance to the planet's center is r. The captain of your spaceship decides to take a closer look, and so she maneuvers your ship to a distance of $r/2$ to the planet's center. (It's still well clear of the planet's surface.) During the movement to get closer, the only forces applied on the spaceship are radial in direction, with zero tangential component. (Radial means straight inward or outward; tangential is tangent to a circle centered on the planet.) The maneuvers use a negligible amount of fuel, so that m_s, the mass of the ship, remains constant. And when you reach $r/2$, the spaceship has a radial velocity component of zero, just as it was at r.

You have three possibilities for when you reach $r/2$:

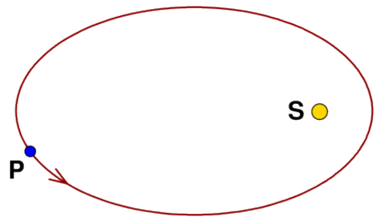
Just right. You have a circular orbit with radius $r/2$, without having to do anything else.

Too fast. You need to slow down the spaceship to remain in a circular orbit with radius $r/2$.

Too slow. You need to speed up the spaceship to remain in a circular orbit with radius $r/2$.

Which one is correct? Produce a calculation that shows why.

2. (30 points) A planet (P) with mass m revolves around a star (S) with mass M, where $M \gg m$, so that the center of mass of the system is almost at the center of the star itself. The orbit is an ellipse, and the star is not at the center of the ellipse, but at one of the focal points. Call the minimum distance between the star and the planet a, and the maximum distance b. The speed of the planet when it is closest to the star is v_a, when it is farthest it's v_b.



(a) Briefly explain why you can use energy conservation in this case, and write down the energy conservation equation that relates the total energy when the planet is closest to the total energy when it is farthest.

(b) You can also use angular momentum, with $\omega_a = v_a/a$ and $\omega_b = v_b/b$. Briefly explain why angular momentum is conserved during the planet's orbit, and write down the equation that relates the angular momentum when the planet is closest to when it is farthest.

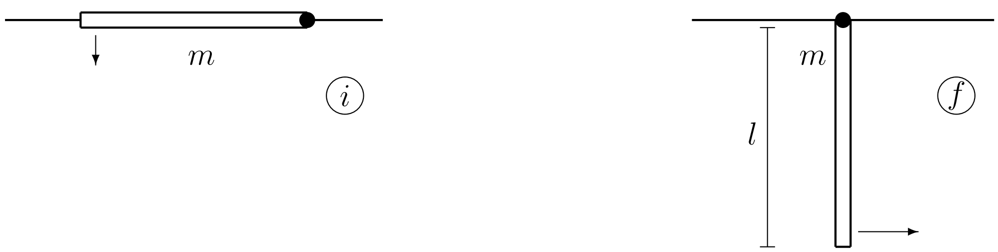
(c) Check if the linear momentum is conserved by looking at the linear momentum of the planet when it is closest and farthest. Explain your result.

(d) Combine your valid conservation equations to eliminate v_b and find an equation for v_a in terms of G, M, a, and b. To simplify your algebra, you can use

$$\frac{\left(\frac{1}{a} - \frac{1}{b}\right)}{1 - \left(\frac{a}{b}\right)^2} = \frac{b}{a(a+b)}$$

(e) Use your result to explain how astronomers can obtain the mass of a star if they can observe the details of the orbit of a smaller mass around this star.

3. (50 points) You have a uniform thin rod attached to the ceiling at one end. Starting from the rod being up against the ceiling, at rest, you let the rod go, and it swings down. The mass of the rod is m, and its length is l. The effects of the drag force on the rod, and the friction at the ceiling, are negligible. In the following questions, when a variable has a subscript f, it refers to what is happening at the bottom of the arc of the rod's swing, when it is positioned completely vertically.



(a) The moment of inertia of a rod rotating around its center of mass is $ml^2/12$. What is its moment of inertia when the axis of rotation is the point of attachment to the ceiling?

(b) Say your y-axis is pointing upward, and the ceiling height is $y = 0$, so that the initial gravitational potential energy is $U_i = 0$. Circle the potential energy of the rod in position f, as it is vertical. Then provide a reason for your choice.

$$-mgl \qquad -\tfrac{1}{2}mgl \qquad -\tfrac{1}{3}mgl \qquad -\tfrac{1}{6}mgl \qquad -\tfrac{1}{12}mgl$$

(c) Make qualitative graphs of θ, the angle of the rod with the ceiling; ω, its angular velocity; and τ, the total torque on the rod. Give the values of θ and τ when $t = t_i = 0$ and $t = t_f$, the time when the rod is vertical. Briefly explain your reasoning.

(d) Can you use linear momentum conservation to find out the angular velocity ω_f? If so, calculate ω_f. If not, explain why.

(e) Can you use energy conservation to find ω_f? If so, calculate ω_f. If not, explain why.

(f) Can you use angular momentum conservation to find ω_f? If so, calculate ω_f. If not, explain why.

Extra Problems (not graded)

4. (0 points) You have a horizontal, disc-shaped platform that rotates around its center. Its mass is M and radius is R. There is no friction at the axis of rotation. On the platform, at radius $r = R/2$, there is a kid with mass $m = M/40$. They both are initially at rest.

(a) The kid steps forward in such a fashion that her r remains constant, propelled by a force $\vec{F}$ due to the friction between the platform and her shoes. As she moves, what is the ratio of the angular acceleration of the kid to the angular acceleration of the platform?

(b) The kid keeps moving in a manner such that her r is constant. What is the ratio of the angular velocity of the kid to the angular velocity of the platform at any instant?

(c) What is the ratio of the total angle covered by the kid since she started moving to the angle rotated through by the platform?

5. (0 points) Astronomers observe a new comet approaching the sun. They obtain the location of the comet relative to the sun, and the velocity of the comet. But they don't have long-term data to directly tell whether the trajectory of the comet is an ellipse or a hyperbola. Still, astronomers can figure it out. After all, an elliptical orbit means that the comet is gravitationally bound to the sun: it can never escape to an infinite distance. But a hyperbolic trajectory extends to infinity: the comet is unbound and must escape the Sun's gravity.

Chose, from the following options, the test that astronomers can apply, using their position and velocity observations, that distinguishes between a bound and an unbound comet. If an option is incorrect, briefly explain why. If it is correct, give the exact inequality they will apply, using the velocity and distance to the sun of the comet, and if needed, data about other objects in the solar system. State whether the inequality indicates a bound or unbound comet.

(a) $L_{\text{comet}} > L_{\text{Earth}}$. (Compare angular momenta.)

(b) $E_{\text{comet}} > 0$. (The total orbital energy of the comet is positive.)

(c) $I_{\text{comet}} > I_{\text{Sun}}$. (Compare moments of inertia.)

(d) $\Delta\vec{p}_{\text{comet}} > \vec{F}_{\text{comet}}$. (Compare the momentum change to the gravity on the comet.)

(e) $\sum \tau_{\text{comet}} > 0$. (The total torque on the comet is positive.)