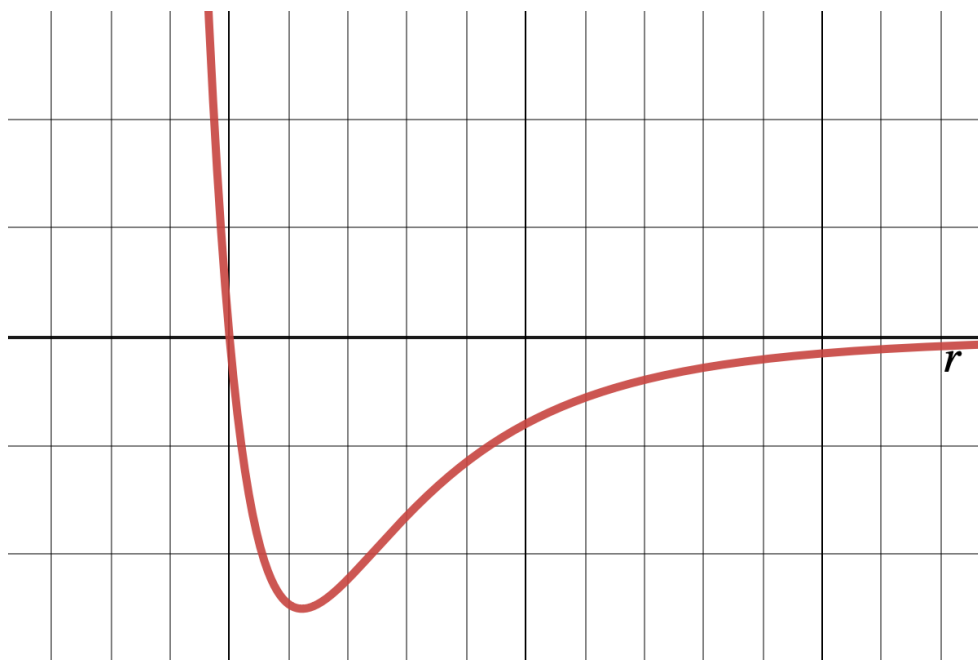


1. (30 points) The following graph gives the potential energy  $U(r)$  due to the interaction between two particles, where  $r$  is the distance between the particles. Notice that  $U$  approaches 0 when  $r$  is very large, and that  $U$  becomes very large when  $r$  approaches 0.

The total energy of a pair of particles is  $E_T = U + K$ , where  $K$  is the kinetic energy due to the relative motion of the particles. There are no forces other than the interaction described by  $U(r)$ .

The two particles are said to be bound to each other if it is impossible for  $r$  to be larger than an upper limit,  $r_{\max}$ . If there is no upper limit—if  $r$  can be arbitrarily large—the particles are unbound.

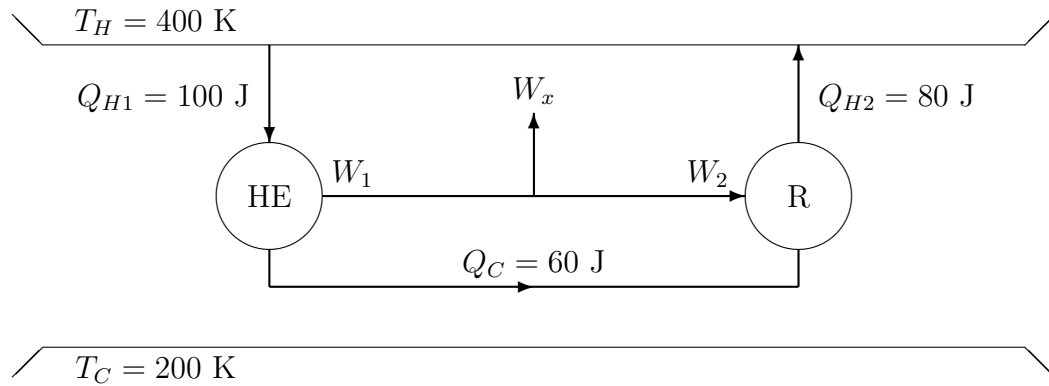


- (a) Say  $E_T > 0$ . Are the particles bound or unbound? Provide an argument. (A visual argument where you draw on the graph above will be fine, if you prefer.)

(b) Say  $E_T < 0$ . Are the particles bound or unbound? Again, provide an argument.

(c) Say you put lots and lots of particles together, where the interaction of each pair of particles is described as above, and where the temperature is such that the *average* total energy  $\bar{E}_T > 0$ . Will this population of particles be in a gaseous state, or will it be in a more condensed state (liquid or solid)? Explain.

**2. (30 points)** An inventor asks you to support his “free energy” device. He has this brilliant idea of using the exhaust heat and work from a heat engine as input to a refrigerator. He says that in each cycle, his heat engine takes in 100 J from a 400 K heat reservoir, and exhausts 60 J. But instead of just dumping that 60 J into a 200 K cold reservoir, he feeds that directly into a refrigerator. He also uses the work output by the heat engine to run the refrigerator, but since the refrigerator only returns 80 J per cycle to the hot reservoir, there is an excess work  $W_x$  that is left over for you to use as you like. You will never pay an electric bill again!

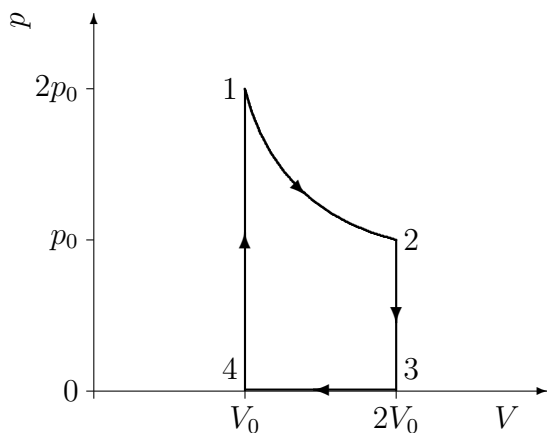


(a) Should you support this invention? Show your calculations.

- (b) If in (a), you found that this invention was workable, are there any values of  $Q_C$  and  $T_C$  for which the invention wouldn't work? If your answer to (a) was "no," are there any values of  $Q_C$  and  $T_C$  for which the invention would work? Explain.

**3. (40 points)** It's physically impossible to have a cold reservoir at absolute zero, but let's see what would happen if such a thing were available.

You have a monatomic ideal gas that goes through the cycle  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$  shown in the diagram. No gas molecules are added or removed during the cycle.



Find everything ( $W$ ,  $\Delta U$ ,  $Q$ ) in terms of  $p_0$  and  $V_0$ .

- (a) The  $1 \rightarrow 2$  part of the cycle takes place at *constant temperature*, so  $T_1 = T_2$ . The area under a constant temperature curve with temperature  $T$  on the  $p$ - $V$  diagram, going from from an initial  $V_i$  to a final  $V_f$ , is

$$NkT \ln \left( \frac{V_f}{V_i} \right)$$

Find the work done on the gas for each step of this cycle:  $W_{1 \rightarrow 2}$ ,  $W_{2 \rightarrow 3}$ ,  $W_{3 \rightarrow 4}$ ,  $W_{4 \rightarrow 1}$ .

- (b) Find the change in thermal energy for each step:  $\Delta U_{1 \rightarrow 2}$ ,  $\Delta U_{2 \rightarrow 3}$ ,  $\Delta U_{3 \rightarrow 4}$ ,  $\Delta U_{4 \rightarrow 1}$ .

(c) Find the heat added to the gas for each step of this cycle:  $Q_{1\rightarrow 2}$ ,  $Q_{2\rightarrow 3}$ ,  $Q_{3\rightarrow 4}$ ,  $Q_{4\rightarrow 1}$ .

(d) Find the total heat input to this gas in one cycle,  $Q_{\text{in}}$ . Also find the total heat removed from the gas,  $Q_{\text{out}}$ , and the total work done on the gas,  $W$ .

(e) What is the efficiency of this heat engine? (Your result should be a number.)

## Extra Problems (not graded)

4. (0 points) You have a heat engine that extracts heat at a rate of 50 kW from a heat reservoir at  $200^{\circ}\text{C}$ , and exhausts 40 kW to a reservoir at  $50^{\circ}\text{C}$ . You then decide that you can use the heat going to waste, so you attach a second heat engine to the exhaust of the first: it takes the 40 kW as input, and finally dumps its own exhaust into a reservoir at room temperature,  $20^{\circ}\text{C}$ .

(a) If the second heat engine is reversible—it produces the least possible amount of exhaust heat—what is the power it produces? What is the rate at which it exhausts heat?

(b) You can think of the combination of the first and second heat engines as a single combined heat engine operating between  $200^{\circ}\text{C}$  and  $20^{\circ}\text{C}$ , producing a total power which is the sum of their outputs. What is the efficiency of this combined heat engine?

5. (0 points) Give an example of an irreversible process—but come up with an example I didn't use in class. In other words, no shattered objects re-forming themselves spontaneously, no billiard balls etc. How can you tell that your example is irreversible?