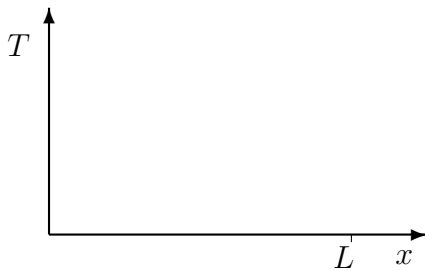


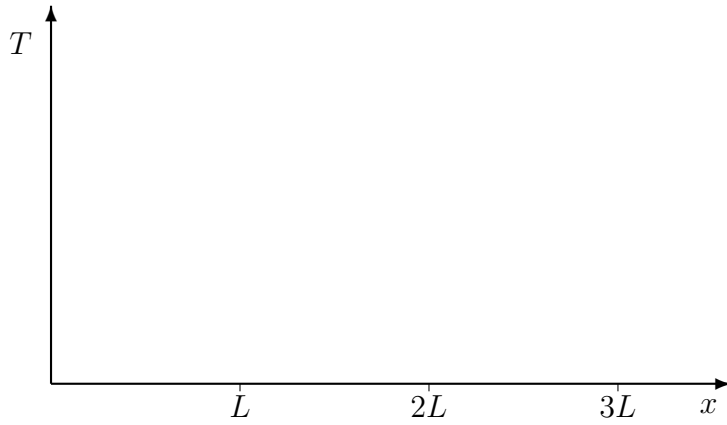
1. (30 points) You have a slab of material through which heat is being conducted at a rate of dQ/dt . The thickness of the material is L , the temperature on one side is T_1 and on the other side it's T_2 .

- (a) What is the temperature in the middle of the slab, a distance $L/2$ from either side?
Hint: The same heat is going through at the same rate through both halves of the slab.

- (b) Make a graph of $T(x)$, the temperature within the slab, with $x = 0$ one side of the slab and $x = L$ the other side. Give your reasoning for the shape you drew.



- (c) You have a double-glazed window between the inside of a room at T_{in} and the outside at T_{out} . The thickness of the the two glass panes and the air trapped is all equal, L . The thermal conductivity of air is less than glass: $k_a < k_g$. Make a graph of $T(x)$, the temperature within the window, with $x = 0$ one side of the slab and $x = 3L$ the other side. Give your reasoning for what you drew.



2. (40 points) If you look up how convection works, you will find $dQ/dt = hA\Delta T$, where A is the surface area of an object, and h is a convection coefficient that depends on the material and its geometric shape. You know how conduction and radiation works.

- (a) You have two cubes made of identical materials, in identical environments, at identical starting temperatures. Cube 1 has a side of length a , cube 2 has $2a$. Find the ratio of the rates at which each cube cools:

$$\frac{\left(\frac{\Delta T_1}{\Delta t}\right)}{\left(\frac{\Delta T_2}{\Delta t}\right)}$$

Note: ΔT refers to the temperature difference with the environment. ΔT_1 and ΔT_2 are *different*—they refer to the change in temperature over time of cubes 1 and 2. Also, you'll need this: for the small temperature ranges in question, $dQ/dt = Q/\Delta t$.

Hint: Your final result should be a number, with no symbols. Cancel things!

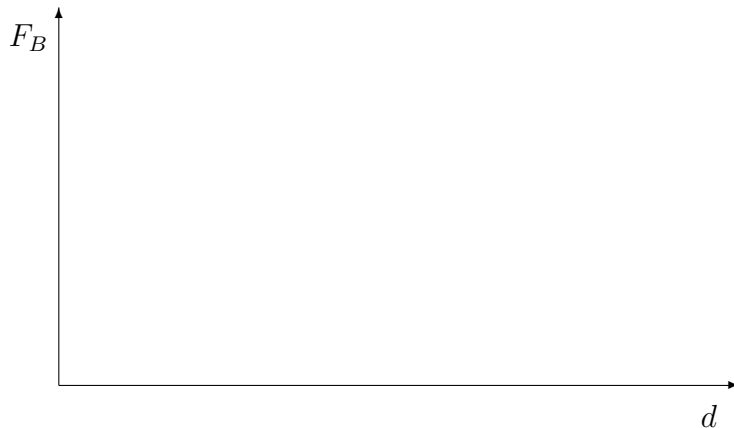
- (b) Use this to predict whether in cold climates, small or large animals will have proportionally thicker coats, and area-reducing adaptations such as smaller external ears. Explain.

3. (30 points) Take a small bubble of air at a depth d below the ocean surface. There are N molecules of air in the bubble, and air is approximated very well as an ideal gas. Let's assume that the bubble is small enough that we can assume a single depth and a single pressure value accurately characterizes the bubble. Let's also assume that the ocean has a constant temperature T at any depth, and that the air is always in thermal equilibrium with the ocean. Use p_{atm} to represent atmospheric pressure and ρ_w to represent the density of water.

(a) Write down an equation for V , the volume of the bubble.

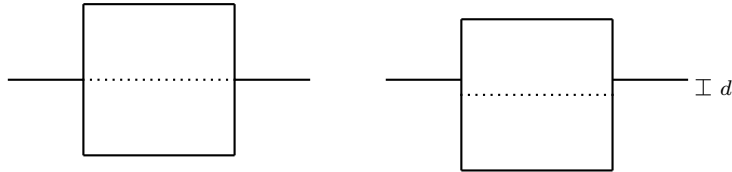
(b) Now write down an equation for the buoyancy force F_B on the bubble.

(c) Make a rough sketch of the buoyancy force versus depth. Make sure the sketch is clear about whether F_B almost at the surface ($d = 0$) is zero, infinite, or a finite value.



Extra Problems (not graded)

4. (0 points) You have a cube with sides a floating in water, with density ρ_w . You mark where the water line is on the cube. Then, you push down the cube by an extra depth d , and let go of the cube. Assume that $d \ll a$ and that you're able to uniformly push the cube down and so forth—in other words, assume that drag forces and any other complications are negligible.



(a) Find what the total force on the cube is at the instant you let it go at depth d .

(b) When $d = 0$, what should the total force be? Check if your answer to (a) behaves accordingly.

(c) Your total force depends on d in a way precisely analogous to how another force we have examined this semester depends on a displacement from equilibrium. Identify this force. Then, using this analogy, qualitatively describe the motion your cube will exhibit once you let it go.

(d) Check your answer to (c) with me. If you have it right, I will ask you another question that will allow you to *quantitatively* describe the motion of the cube.

5. (0 points) If a space traveler were to step out into the vacuum of empty space, far away from all stars and other sorts of heat, she would be in an environment close to absolute zero: the 3 K of the cosmic microwave background radiation, in fact. To calculate if she would freeze to death, you gather the following information. The thermal conductivity of her clothes: 0.05 W/m·K. Thickness of her clothes: 0.010 m. Her surface area: 1.8 m². The emissivity of her clothes surface: 0.5. Stefan-Boltzmann constant: 5.67×10^{-8} W/m²·K⁴. The surface temperature of her clothes: 35°C. The specific heat of mammalian tissue: 3400 J/kg·K. Her mass: 56.0 kg. Rate of thermal energy generation by a human body at rest: 100 W. (Not all these values are necessarily useful!)

(a) Calculate the rate the astronaut absorbs heat from the microwave background.

(b) Calculate her *net* rate of heat loss.

(c) Calculate the rate at which her body temperature will drop, which is $dT/dt \approx \Delta T/\Delta t$ for small time intervals.

(d) How long will it take for her to cool by 1 K?