Solutions to Exam 1; Phys 185

1. (60 points) You set up an experiment to measure the coefficient of kinetic friction between the cart and track we've been using in the lab. You lay the track flat, set a *single* light gate close to one end of the track, and send a cart with a sail through the light gate. You practice until you get the initial velocity of the cart such that the cart slows down and stops close to the other end of the track without falling off. For each trial, then, you will have three quantities you can measure: l (the length of the sail), d (the distance between the light gate and the trailing end of the sail at the point where the cart stops), and $\tau = t_2 - t_1$ (the time interval between the sail entering the light gate at t_1 and exiting the light gate at t_2).

(a) If the sail length l is small, and you're using a low friction track, you can assume that friction will have a negligible effect during the short distance through the light gate. You can then determine the initial velocity of your cart by assuming that the cart has a constant velocity through the light gate, and that friction starts taking effect only after the cart is through. Under these assumptions, find an equation for μ_k in terms of quantities that you know or can measure.

Answer: The forces acting on the cart are weight, normal force, and friction. Therefore,

$$\sum F_x = w_x + n_x + f_{kx} = 0 + 0 - f_k = -\mu_k n = ma_x$$

$$\sum F_y = w_y + n_y + f_{ky} = -mg + n + 0 = 0$$

Combining these, we get

$$a_x = -\mu_k g$$

If we can assume constant velocity within the gate, the velocity exiting the gate is $v_{2x} = l/\tau$. The time to come to a stop will then be

$$0 = \frac{l}{\tau} - \mu_k g t \quad \Rightarrow \quad t = \frac{l}{\mu_k g \tau}$$

We also know the distance to come to a halt, d. Therefore

$$d = 0 + \frac{l}{t}\frac{l}{\mu_k g \tau} - \frac{1}{2}\mu_k g \left(\frac{l}{\mu_k g \tau}\right)^2 = \frac{l^2}{2\mu_k g \tau^2}$$

Solving, we get

$$\mu_k = \frac{l^2}{2gd\tau^2}$$

for the friction coefficient.

(b) But there *is* friction as the cart moves through the light gate, and the velocity coming out of the light gate should be slightly less than what you calculated in (a). Now do a more proper calculation, accounting for the effect of friction as the cart goes through the light gate. You will end up with a quadratic equation for μ_k; it is sufficient if you find this equation (you don't have to solve it for μ_k, though that isn't too difficult).

Answer: Friction means that the velocity exiting the gate, v_{2x}, is smaller than the entry velocity v_{1x}. The forces are the same as before, so $a_x = -\mu_k g$ remains true. Therefore, for the passage through the light gate,

$$v_{2x} = v1x - \mu_k g\tau \quad \text{and} \quad l = v_{1x}\tau - \tfrac{1}{2}\tau^2$$

We need v_{2x}, the exit velocity from the light gate. To get there, we first need to solve the second equation for v_{1x}:

$$v_{1x} = \frac{1}{\tau}\left(l + \tfrac{1}{2}\tau^2\right) \quad \Rightarrow \quad v_{2x} = \frac{l}{\tau} + \tfrac{1}{2}\mu_k g\tau - \mu_k g\tau = \frac{l}{\tau} - \tfrac{1}{2}\mu_k g\tau$$

This is smaller than l/τ, the velocity out of the gate used in part (a), which is as it should be.

Now, we repeat the calculation of part (a), but with this different initial velocity. The result will just be replacing l/τ with the corrected value, so

$$\mu_k = \frac{1}{2gd}\left(\frac{l}{\tau} - \tfrac{1}{2}\mu_k g\tau\right)^2$$

This equation has μ_k on both sides. To solve it, we need to do the square, and arrange everything into a quadratic equation:

$$2\mu_k gd = \frac{l^2}{\tau^2} - l\mu_k g + \tfrac{1}{4}\mu_k^2 g^2\tau^2 \quad \Rightarrow \quad \left[\tfrac{1}{4}g^2\tau^2\right]\mu_k^2 - \left[(l+2d)g\right]\mu_k + \left[\frac{l^2}{\tau^2}\right] = 0$$

The solution we want is, after some simplification,

$$\mu_k = \frac{2}{g\tau^2}\left(l + 2d - 2\sqrt{d(d+l)}\right)$$

Leaving it as a quadratic equation is fine, though.

(c) Reasonable values in the lab might be $l = 0.15\,\text{m}$, $d = 1.60\,\text{m}$, and $\tau = 0.600\,\text{s}$. Calculate μ_k according to your result in (a) and in (b). Is the approximation in (a) valid?

Answer: The approximate answer from (a) is, when you put in the numbers, $\mu_k = 0.0020$. The more accurate answer from (b) is $\mu_k = 0.0019$.

These are not very different, as we might expect from a low-friction track. But it is, nonetheless, a 5% error. If you want any accuracy in determining μ_k, you do need the more complicated equation.

2. (40 points) The drag equation we worked with is an approximation useful for low-density gases, such as air, under everyday conditions. Then, we had $D = (\text{constants})v^2$. But under different conditions, the speed-dependence of drag can be different. In general, $D = f(v)$, where $f(v)$ is a function of speed, and the direction of $\vec{D}$ is always opposite the velocity $\vec{v}$.

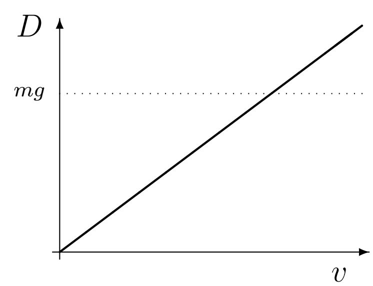
The following four graphs represent four different cases for $D = f(v)$. With each of these four drag forces, you release an object from rest, and let it fall under the influence of only gravity and that drag force. In each of these cases, the object will do one of:

i. Always speed up until it reaches a downward terminal velocity.

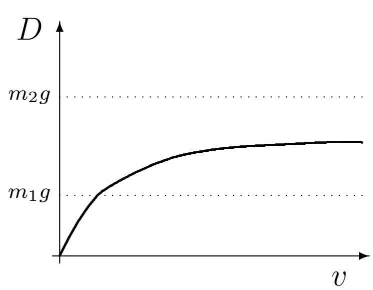
ii. Never reach a downward terminal velocity.

iii. It may or may not reach a terminal velocity, depending on a condition.

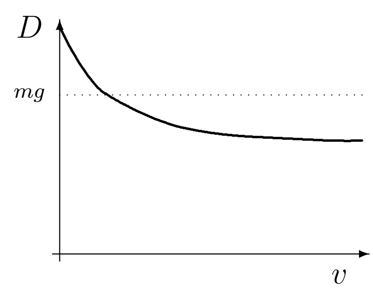
For each of the graphs, identify what happens and give an explanation why. If you choose option (iii) for a graph, tell me the condition for reaching terminal velocity.



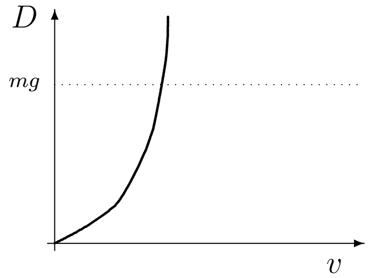
Answer: An object approaches terminal velocity because the drag force on it increases with its speed, which means that an increasing drag force acts opposite to its weight, coming closer and closer to canceling out the weight entirely. In this case, the drag force magnitude D increases as v increases, and $D = mg$ will happen at some v, regardless of the mass m. So (i) will be correct: speed up until a terminal velocity.



Answer: This one is (iii). If m_1g is smaller than the value where D levels off, $D = m_1g$ will happen, and that will be a terminal velocity. But if the weight m_2g is too large, $D = m_2g$ will not be possible, and there will be no terminal velocity.



Answer: This is (ii). It may look like $D = mg$ is possible, hence there is a terminal velocity. But you will never reach it if you start at $v_y = 0$. In this case, you will get $\sum F_y > 0$, and you will accelerate *upwards,* and your velocity will be in the wrong direction.



Answer: This is another (i). As the object falls, it will speed up and D will grow toward mg. In this graph, there is a maximum possible v_T regardless of the mass m, which is peculiar, but there still is a v_T.