Solutions to Exam 2; Phys 185

1. (20 points) You can use your centripetal motion apparatus from Lab 4 to determine the spring constant k of the spring attached to the mass that goes in a circle. Using your equipment, you can measure the mass m, the period T, the radius of the circle r, and the equilibrium (unstretched) length of the spring l. Find an equation for k.

Answer: The total force must produce uniform circular motion. When the mass is spinning at the proper radius, the vertical forces are tension and weight, which cancel each other out. The horizontal force is just the spring force, and the spring is stretched by an amount of $r - l$. Therefore

$$\sum F_x = k(r-l) = m\frac{v^2}{r} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 mr}{T^2} \qquad \Rightarrow \qquad k = \frac{4\pi^2 mr}{(r-l)T^2}$$

2. (40 points) You have a pendulum that consists of a mass m attached to a string with length l. Let θ represent the angle between the string and the ceiling. You start with the pendulum up against the ceiling ($\theta = 0$) and at rest. You then let the pendulum go, and it starts swinging back and forth. The mass of the string, drag, and friction are negligible.

(a) Find what the angular acceleration α is at an arbitrary angle θ.

Answer: Use the connection between the torque τ and α. The tension force is always directed toward the axis of rotation, so it produces no torque. Only the weight produces torque. The mass is a point particle, so

$$\sum \tau = F_\perp l = mgl\cos\theta = I\alpha \quad \text{and} \quad I = ml^2 \qquad \Rightarrow \qquad \alpha = \frac{mgl\cos\theta}{ml^2} = \frac{g}{l}\cos\theta$$

Note that α is maximum at the top, and is zero at the very bottom, as it should be.

(b) Find what the speed v of the mass is at an arbitrary angle θ. (I'll provide trig help!)

Answer: Use energy conservation. At first, the mass is at rest, so $K_i = 0$. If you measure y from the ceiling, $U_i = mg0 = 0$. So the total energy is 0. And with nonconservative forces negligible, $E_{\text{loss}} = 0$. At angle θ we will have $y = -l\sin\theta$, so the total energy will be

$$K_f + U_f = \tfrac{1}{2}mv^2 - mgl\sin\theta = 0 \qquad \Rightarrow \qquad v = \sqrt{2gl\sin\theta}$$

This v starts from zero at the top, and becomes a maximum at the bottom, as you would expect.

3. (40 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You perform a completely elastic collision between the first cart, with mass m and initial velocity $v_{1i} = v$, with a target cart initially at rest with mass qm (q is an arbitrary number, $0 \le q \le \infty$).

(a) Find v_{1f} and v_{2f}, the velocities of each cart after the elastic collision. For full credit, do the algebra and get your results. However, with a 10-point cost, you can also pick between the following options. If you do that, also calculate v_{1f} and explain your choice.

(i) $v_{2f} = \left(\frac{2}{q+1}\right) v$

(ii) $v_{2f} = \left(\frac{qm}{2q+1}\right) v$

(iii) $v_{2f} = \left(\frac{q}{2m}\right) v$

(iv) $v_{2f} = \left(\frac{1+m}{q+m}\right) v$

(v) $v_{2f} = \left(\frac{qm}{2}\right) v$

Answer: Momentum conservation gives

$$mv = mv_{1f} + qmv_{2f} \qquad \Rightarrow \qquad v_{1f} = v - qv_{2f}$$

An elastic collision means that $E_{\text{loss}} = 0$, so energy conservation gives

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}qmv_{2f}^2 = \tfrac{1}{2}m(v - qv_{2f})^2 + \tfrac{1}{2}qmv_{2f}^2$$

Expanding the square as $(v - qv_{2f})^2 = v^2 - 2qvv_{2f} + q^2v_{2f}^2$ and doing some cancellations, we get

$$0 = (\tfrac{1}{2}q^2m + \tfrac{1}{2}qm)v_{2f}^2 - qmvv_{2f} = v_{2f}\left[(\tfrac{1}{2}q^2m + \tfrac{1}{2}qm)v_{2f} - qmv\right]$$

One solution is $v_{2f} = 0$, but that indicates no collision. The other solution is when

$$(\tfrac{1}{2}q^2m + \tfrac{1}{2}qm)v_{2f} - qmv = 0 \qquad \Rightarrow \qquad v_{2f} = \frac{mqv}{\frac{1}{2}mq(q+1)} = \left(\frac{2}{q+1}\right)v$$

The other final velocity is then

$$v_{1f} = v - qv_{2f} = \left(1 - \frac{2q}{q+1}\right)v = \left(\frac{1-q}{q+1}\right)v$$

If you picked among the given answers, (i) was correct.

(b) Check your results. Find what v_{1f} and v_{2f} are for the cases $q = 0$, $q = 1$, and $q \to \infty$ (when q is very large). What sort of motion does each case describe? Do these make sense? (Feel free to check with me about any of your results.)

Answer: When $q = 0$, $v_{1f} = v$ and $v_{2f} = 0/0$. The target is massless, so it can do anything; the first cart is unaffected and goes through with its initial velocity.

When $q = 1$, $v_{1f} = 0$ and $v_{2f} = v$. The masses are equal, and this result is one that you have seen before.

When $q \to \infty$, $v_{1f} = -v$ and $v_{2f} = 0$. The target is immovable, and doesn't move, and the first cart bounces back at an equal speed but opposite direction.