Solutions to Exam 4; Phys 185

1. (30 points) You have two objects that are identical in almost every respect. One of them is lower (closer to the Earth) and has a smaller drag coefficient. The other is immediately above, and has a larger drag coefficient. The objects are tied to each other by a practically massless, extremely thin string.

(a) You put the objects in a vacuum chamber close to the surface of the Earth and let go, observing free fall. Draw a diagram of the objects and show the forces on each object. Choose the correct answer about the string tension, and explain your choice or provide a calculation:

- **(i)** $T = 0$ **(no tension)**
- (ii) $T > 0$ (the string is taut)
- (iii) $T < 0$ (the string goes slack and the objects get closer)

Answer: Each object will have its weight acting on it, downward. And each object will also have the tension in the string acting on it. The tension magnitude T is the same, but it acts downward on the upper object, and upward on the lower object. The objects in free fall will have the same acceleration, and since their masses are the same, their forces will add up to the same total $\sum F_y$. Therefore

$$-T - mg = T - mg \qquad \Rightarrow \qquad T = 0$$

No tension!

(b) You put the objects in the air close to the surface of the Earth. Draw a diagram of the objects and show the forces on each object. Choose the correct answer about the string tension, and explain your choice or provide a calculation:

- (i) $T = 0$ (no tension)
- **(ii)** $T > 0$ **(the string is taut)**
- (iii) $T < 0$ (the string goes slack and the objects get closer)

Answer: Now there will also be drag on both objects, and since the drag coefficient is larger for the top object, $D_{\text{top}} > D_{\text{bottom}}$. The accelerations of both objects, and therefore $\sum F_y$ for each object will still be the same.

$$-T - mg + D_{\text{top}} = T - mg + D_{\text{bottom}} \qquad \Rightarrow \qquad T = \tfrac{1}{2}(D_{\text{top}} - D_{\text{bottom}}) > 0$$

There must be tension in the string to keep the objects falling together.

(c) You put the objects in outer space. The distance between the lower object and the center of the earth is $2R_E$, where R_E is the radius of the Earth. The distance between the higher object and the center of the earth is $3R_E$. Neither object is in orbit around the Earth; they fall straight down. Draw a diagram of the objects and show the forces on each object. Choose the correct answer about the string tension, and explain your choice or provide a calculation:

(i) $T = 0$ (no tension)

(ii) $T > 0$ **(the string is taut)**

(iii) $T < 0$ (the string goes slack and the objects get closer)

Answer: In this case, the gravitational force on the top (far away) object is weaker than the gravity on the bottom. The accelerations of both objects, and therefore $\sum F_y$ for each object will still be the same.

$$-T - F_{\text{G,top}} = T - F_{\text{G,bottom}} \qquad \Rightarrow \qquad T = \tfrac{1}{2}(F_{\text{G,bottom}} - F_{\text{G,top}}) > 0$$

2. (20 points) You do a collision experiment such as Labs 6 and 7, but with expensive equipment that makes friction and drag forces completely negligible. Cart 1 has mass m and heads toward the right with initial velocity $2v$. Cart 2 has mass $2m$ and heads toward the left with initial velocity $-v$. The carts then collide.

(a) If the carts stick together after the collision, what are the final velocities of the carts?

Answer: The total linear momentum is conserved, but the total momentum is zero!

$$m(2v) - (2m)v = 0 = (2m + m)v_f \qquad \Rightarrow \qquad v_f = 0$$

Nothing moves.

(b) If the collision is elastic, what are the final velocities of the carts?

Answer: The total momentum remains zero, so to conserve energy as well, the cats must bounce back with the reverse of their initial velocities.

$$0 = mv_{1f} + (2m)v_{2f} \qquad \Rightarrow \qquad v_{1f} = -2v_{2f}$$

Using this result in energy conservation,

$$\tfrac{1}{2}m(2v)^2 + \tfrac{1}{2}(2m)v^2 = \tfrac{1}{2}m(-2v_{2f})^2 + \tfrac{1}{2}(2m)v_{2f}^2 \quad \Rightarrow \quad v_{2f}^2 = v^2$$

Take the positive solution: $v_{2f} = v$. And then, $v_{1f} = -2v$.

3. (50 points) You have a spherical object falling through the air at terminal velocity. The air has a temperature of 283 K and a density of 1.20 kg/m^3. The sphere has a density of 3000 kg/m^3, a radius of 0.500 m, and a drag coefficient $C_D = 0.500$. The sphere is also a deep black, with $e = 1$ for all wavelengths of radiation. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/(m^4{\cdot}K^4)}$. The volume of a sphere is $\frac{4}{3}\pi r^3$. The surface area of a sphere is $4\pi r^2$. The area of a circle is πr^2.

(a) As the sphere falls under the influence of the drag force, there is an energy loss. Say the sphere falls for 1.00 s. What is E_{loss} during that time?

Answer: Either calculate the work done by the drag force, or use energy conservation:

$$\tfrac{1}{2}mv_T^2 + mgh = \tfrac{1}{2}mv_T^2 + 0 + E_{\mathsf{loss}}$$

whre h is the height the sphere falls in one second. The kinetic energy does not change, since the sphere is always at terminal velocity. Therefore

$$E_{\mathsf{loss}} = mgh \qquad \text{with} \qquad h = v_T(1\,\mathrm{s})$$

We need

$$v_T = \sqrt{\frac{2mg}{\rho_{\mathsf{air}} A C_D}} = \sqrt{\frac{2\rho_{\mathsf{sph}}\frac{4}{3}\pi r^3 g}{\rho_{\mathsf{air}}\pi r^2 C_D}} = 255.6\,\mathrm{m/s} \quad \text{and} \quad h = 255.6\,\mathrm{m}$$

And therefore

$$E_{\mathsf{loss}} = 3.93 \times 10^6\ \mathrm{J}$$

(b) What is the *rate* of energy loss during the fall of the sphere? (This is trivial given your result in (a): do not attempt a calculation!)

Answer: Since what we just found was the energy lost for one second, the energy loss per second is

$$\frac{d}{dt}E_{\mathsf{loss}} = 3.93 \times 10^6\ \mathrm{W}$$

(c) Is the lost energy transferred as heat or as work?

Answer: Heat. There's no net compression or expansion of gases or pushing anything around going on, so there is no work being done.

(d) The lost energy is transferred both to the sphere and the air around it, but we can make the approximation that all of the energy is transferred to only one: the sphere or the air. Which one? That $\rho_{\mathsf{sphere}} \gg \rho_{\mathsf{air}}$ should be relevant to your answer.

Answer: Sphere. There is a lot more mass associated with the sphere than the thin layer of air surrounding it. If it were a vacuum, *all* the energy would go to the sphere.

(e) Let's say the sphere has been falling for long enough that its surface temperature is no longer changing. Which of the following will help you determine the surface temperature?

(i) μ_k between the air and the sphere will remain constant.

(ii) L for the sphere will be conserved once T becomes constant.

(iii) Heating and cooling produces an exponential $\Delta T = \Delta T_0 \, e^{-t/\tau}$.

(iv) For the sphere, $(\frac{dQ}{dt})_{\textsf{out}} = (\frac{dQ}{dt})_{\textsf{in}}$.

(v) $p_{\textsf{atmosphere}}$ changes with h, but slowly enough that it can be assumed constant.

(f) Conduction, convection, and radiation will all be involved. But assume that radiation is the only thing you need to consider, which is accurate enough for a rough calculation. Find the surface temperature of the falling sphere.

Answer: We know the heating rate: $(\frac{dQ}{dt})_{\textsf{in}} = 3.93 \times 10^6\,\mathrm{W}$. The outgoing heat transfer is due to radiation:

$$\left(\frac{dQ}{dt}\right)_{\textsf{out}} = e\sigma A \Delta(T^4) = e\sigma(4\pi r^2)(T^4_{\textsf{surf}} - T^4_{\textsf{air}})$$

Therefore

$$T_{\textsf{surf}} = \left[\frac{(dQ/dt)_{\textsf{in}}}{e\sigma(4\pi r^2)} - T^4_{\textsf{air}}\right]^{1/4} = 2168\,\mathrm{K}$$

Hot!