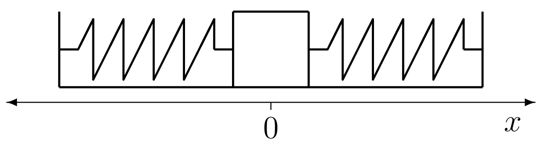
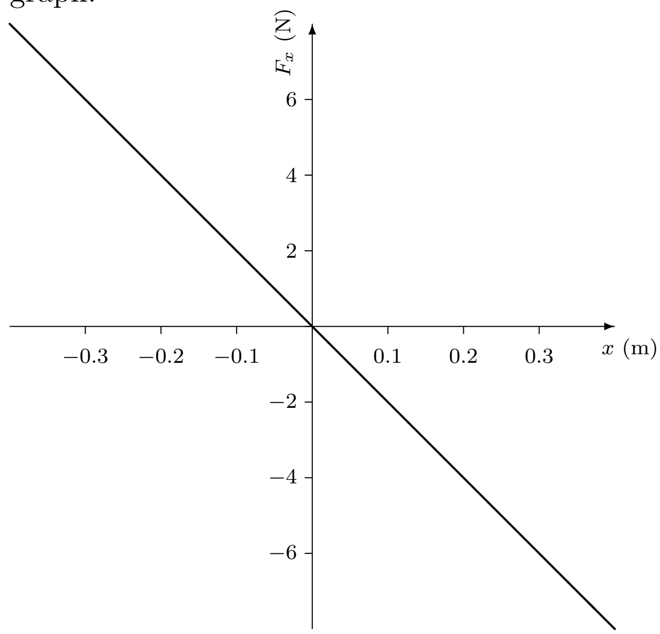
Solutions to Assignment 1; Phys 186

1. (30 points) You have a 0.32 kg object attached to two identical springs, each with spring constant 10.0 N/m, at each end as shown in the diagram. The mass oscillates back and forth horizontally on a frictionless surface.



(a) Using the axes given, draw a graph of the total force on the mass F_x vs. the displacement from equilibrium x. Take care to indicate the force direction with appropriately positive or negative quantities. Write in the appropriate numbers on the tick marks on the F_x axis on your graph.



(b) The oscillations of a single spring have an angular frequency $\omega = \sqrt{k/m}$. What is the angular frequency for the oscillations of this double spring setup? Explain.

Answer: From the graph you can see that this setup is equivalent to a single spring with an effective spring constant $2k = 20.0$ N/m. Therefore

$$\omega = \sqrt{\frac{2k}{m}} = 7.91 \text{ Hz}$$

2. (30 points) You're given a spring, a known mass m_0, and an unknown mass m_1. The only measuring device you have is a stopwatch. Describe an experiment you would design in order to determine m_1. Provide an equation that expresses m_1 in terms of m_0 and quantities you can measure with your stopwatch.

Answer: With a stopwatch, the only thing about the spiring-and-mass systems we can measure is the periods.

The period of oscillations will depend on the mass. So we first measure the period T_0 for oscillations with the known mass. (To reduce error, we probably should measure how long $N \geq 100$ oscillations take, and find the period by dividing the reading on the stopwatch by N.) We use the same procedure to measure T_1. Then, notice that the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}$$

Therefore, k will cancel out if we take the ratio of the two periods:

$$\frac{T_1}{T_0} = \sqrt{\frac{m_1}{m_2}} \quad \Rightarrow \quad m_1 = \left(\frac{T_1}{T_0}\right)^2 m_0$$

3. (20 points) We characterize waves by their frequency, wavelength, and amplitude. Audible sound has a frequency range of 20 Hz to 20 kHz, wavelengths between 1.7 cm and 17 m, and a minimum intensity of 10^{-12} W/m^2. *Ultrasound* can be used for medical imaging, where it can resolve structures with sizes considerably less than 1 cm. The "ultra" in ultrasound must therefore refer to higher than audible frequency, wavelength, or amplitude—which one? Explain. For the range of values in question, you can take the speed of sound to be constant.

Answer: Since medical imaging should see detail with sizes less than 1 cm, we need sound with wavelengths *less* than audible wavelengths which start at 1.7 cm. Since $v = \lambda f$, and v is constant, that means that wavelength and frequency are inversely related. So smaller than audible wavelengths corresponds to *higher than audible frequency.* Amplitude is irrelevant.

4. (20 points) You have a radio beacon set up in outer space, broadcasting with equal and constant power in all directions.

(a) *Qualitatively* sketch the following graphs of the broadcast waves' intensity, amplitude, frequency, and wave speed vs. the distance r from the beacon.

(b) Give the r-dependence of all four variables. The answer for each should be one of r^2, r^1, r^0 (constant), r^{-1}, or r^{-2}. (*Note:* the symbol "$\propto$" means "proportional to.")

Answer:

$$I \propto r^{-2}$$

$$A \propto r^{-1} \text{ since } I \propto A^2$$

$$f \propto r^0$$

$$v \propto r^0$$

with corresponding graphs.

Extra Problems (not graded)

5. (0 points) You have a mass m attached to a frictionless spring with spring constant k, and you set the mass oscillating. In the following list of variables that might describe the resulting motion, draw a circle around those that depend on the mass m:

amplitude, wavelength, period, phase, diffraction

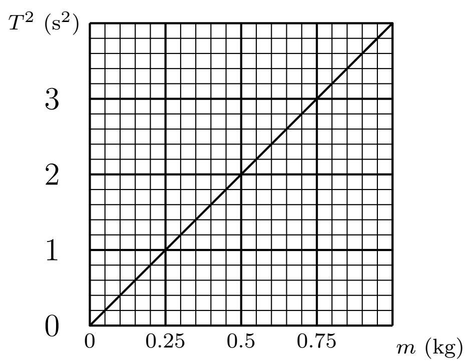
Sketch a graph of this dependence (with m on the horizontal axis) for each variable you circle.

Answer: The amplitude and phase have to do with initial conditions; they do not depend on the mass. Oscillations are not waves; they have no wavelength. Diffraction also applies to waves only.

The period is $T = 2\pi\sqrt{m/k}$, therefore $T \propto \sqrt{m}$. The graph should show a square root dependence.

6. (0 points)

(a) Imagine you hung masses from the end of an ideal spring, with zero mass and no limits on extension or compression without deformation. The spring constant is $k = 10.0$ N/m. You use masses m varying between 0.00 kg and 1.00 kg, and set each mass oscillating, measuring the period T of the resulting motion. What would your predicted graph for the square of the period (T^2) versus mass (m) look like? Put in appropriate numbers on the axes.



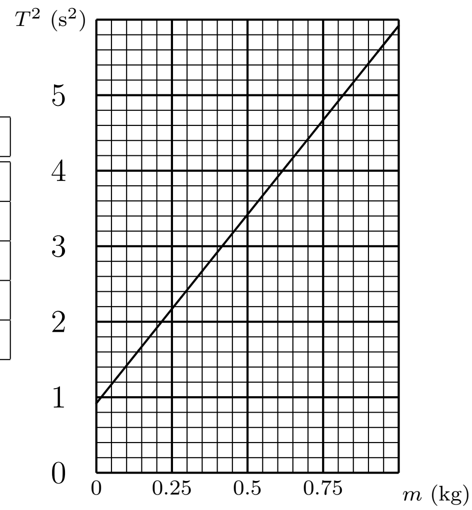
Answer: Since $T = 2\pi\sqrt{m/k}$, its square goes like

$$T^2 = \left(\frac{4\pi^2}{k}\right) m = (3.95\ \mathrm{s^2/kg})\, m$$

This is a linear dependence, with a line going through the origin.

(b) You now get a real spring, and make the following measurements of period for different masses. Make the graph and find k. (*Hint:* Use the slope.)

m (kg)	T (s)	T^2 (s^2)
0.00	0.99	0.98
0.25	1.49	2.22
0.50	1.86	3.46
0.75	2.17	4.71
1.00	2.43	5.90



Answer: The slope of the graph is $4\pi^2/k$. Therefore, figuring out the slope and solving for k, we get 8.0 N/m.

(c) From your graphs, what is the difference between the behavior of the real and ideal springs? Which of the following equations, with an extra parameter ξ, do you think best describes the real spring's period? (*Hint:* $\xi = 0$ for an ideal spring, $\xi \neq 0$ for a real spring.)

$$T = 2\pi\sqrt{\frac{m+\xi}{k}}$$

Answer: This is the only equation that gives $T^2 \neq 0$ when $m = 0$. The difference between the real and ideal springs is that the real spring graph does not go through the origin—at $m = 0$, with no extra masses hanging, the real spring still behaves as if there is some mass hanging on it.

(d) What property of the real spring, ignored in an ideal spring, do you think gives rise to the parameter $\xi \neq 0$? Explain your reasoning.

Answer: The extra mass is due to the mass of the spring itself, which is neglected for an ideal spring.