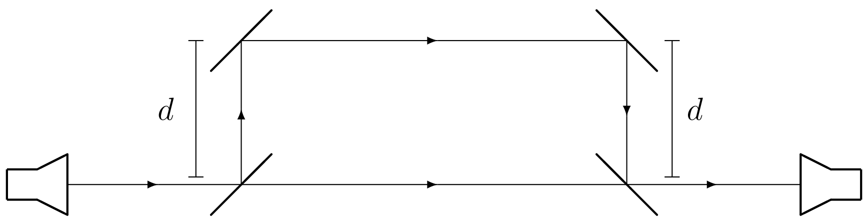
Solutions to Assignment 2; Phys 186

1. (30 points) You have the following interferometer set up in a lab:



You have a microwave source on the left, and a microwave detector on the right. The microwaves have a wavelength of 3.00 cm.The two diagonal lines on top stand for fully silvered mirrors; the two diagonal lines lined up with the source and detector are half-silvered mirrors that reflect and transmit half the incident microwave intensity. The distance between the pairs of fully silvered and half-silvered mirrors is $d = 10.0$ cm. All other distances are irrelevant to the problem.

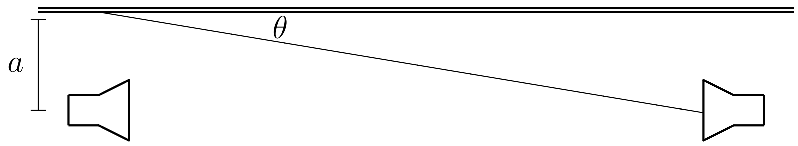
In other words, you split the microwave beam in two, and make one part travel a longer distance before joining the original beam.

Now you start increasing d. Find the next two values of d for which you get *constructive* interference (an intensity maximum at the detector).

Answer: The path length for the two beams differs by $l_1 - l_2 = 2d$. The condition for constructive interference is $l_1 - l_2 = m\lambda$. At first, $2d$ is not an integer multiple of λ. But if we move the mirrors so that $2d = 21$ cm, this is a multiple of λ. This means $d = 10.5$ cm. The next point of constructive interference will come at $2d = 21 + 3 = 24$ cm, which happens when $d = 12$ cm.

If you really wanted to account for everything, you would also deal with the half-wavelength phase shifts that come with each reflection. But these add up to an integer multiple of a wavelength, and hence do not affect the interference at all.

2. (30 points) You have the following set up in a lab:

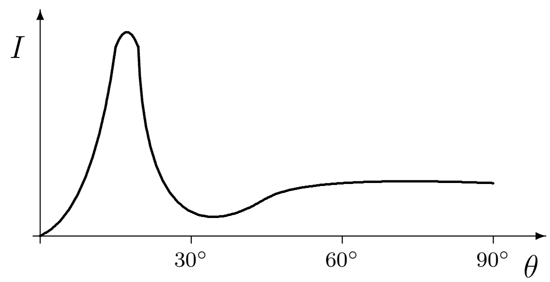


You have a microwave source on the left, and a microwave detector on the right. The microwaves have a wavelength of 3.00 cm. The double lines above represent an oven liner, which reflects the microwaves. The detector can be moved through an angle $0 < \theta < 90^\circ$, as shown. The source is a distance $a = 2.75\,\text{cm}$ to the mirror.

In any textbook, you will see that the effect of a mirror is equivalent to having an identical second source of a light wave located a distance a on the *other side* of the mirror; for example, Figure 25.38 in the online textbook I listed in the syllabus. The oven liner will work the same way with microwaves. And then, there is the extra phase shift to consider, just the same as in Part 2 of your Lab 3.

Sketch a qualitative graph of how the detected intensity will vary with the angle θ, much like Part 3 of your Lab 3. Explain your reasoning. You will be able to find the angles for some maxima or minima of intensity. Calculate these angles. (This will be hard to do if you don't talk to me and ask me questions as you are working on this problem!)

Answer: With the second identical source behind the mirror, this becomes the same situation as a double-slit, with one difference: because of the phase shift with reflection, the locations of the constructive and destructive interference points gets switched. Therefore $\sin\theta_m = m\lambda/d$ will describe the angles of intensity minima, not maxima. The separation between the sources is $d = 2a = 5.50\,\text{cm}$. That's about the same as in your Lab 3 Part 3; similarly, you will only be able to get angles for $m = 0, \pm 1$. These are $\theta_0 = 0$ and $\theta_{\pm 1} = \pm\sin^{-1}(3/5.5) = \pm 33^\circ$. Sketch the graph for I vs $0 < \theta < 90^\circ$ with I minima at these values. While I will approach zero at $\theta = 0$, because the path lengths become identical, I will not be zero at the 33° minimum.



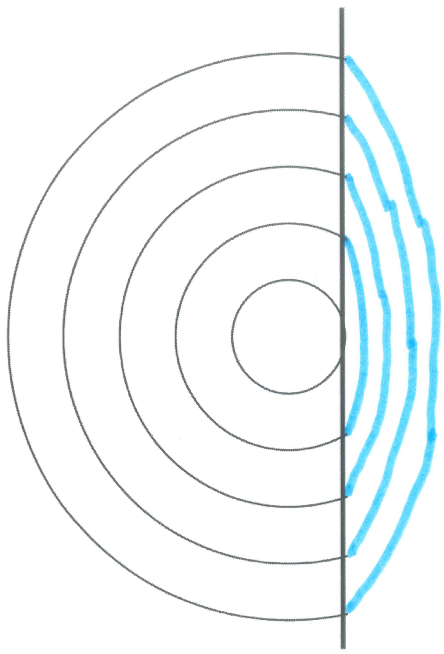
3. (20 points) Say you have a diffraction grating with with slit spacings just slightly larger than visible light wavelengths. When you shine a narrow beam of white light through this grating onto a screen, you will see a bright white spot straight down the middle, and after some darkness, a rainbow pattern on either side. Will the rainbow pattern have red light closest to the white in the middle, with violet farthest away, or violet closest to the middle, with red farthest away? Explain, using the appropriate equations.

Answer: The diffraction grating equation that gives the locations of the intensity maxima is $\sin\theta_m = m\lambda/d$. The central maximum corresponds to $m = 0$, which leads to $\theta = 0$, regardless of λ. So the white light through the center will not get separated into different wavelengths corresponding to different colors.

The rainbow pattern at $m = 1$ is described by $\theta_1 = \sin^{-1}(\lambda/d)$. This means that small λ corresponds to small angles, and large λ to larger angles. The shortest visible wavelength, therefore, will end up with the smallest angle, and therefore the spot on the screen it produces will be closest to the central maximum: violet closest to the middle and red the farthest away.

4. (20 points) You have water waves on the surface of a lake, with wavefronts that spread as concentric circles from a central source, traveling at a constant speed v_{deep}. The straight line indicates a boundary where the lake bottom suddenly steps up, so the waves enter a shallow region where $v_{\text{shallow}} < v_{\text{deep}}$.

(a) Draw how the wavefronts will look in the shallow part of the lake.



(b) Briefly explain your reasoning.

Answer: In the shallow region, the frequency will remain the same, so the wavelength must become smaller. The wavefronts will extend into the shallow region without new wavefronts being created or destroyed at the interface, so each circle will have to be completed.

Extra Problems (not graded)

5. (30 points) Your microwave optics lab included a double-slit experiment where $d = 5.5\,\text{cm}$ and $\lambda = 3.0\,\text{cm}$. You were only able to observe the $m = 0, \pm 1$ peaks of intensity.

(a) With a different d, you might have been able to observe the $m = \pm 2$ peaks. Find a d value that would allow you to see these peaks but not $m = \pm 3$. Make reasonable assumptions about what you need to be able to see peaks with the setup we used.

Answer: To be able to see the $m = \pm 2$ peaks, $\sin\theta_2 = 2\lambda/d$ must have a solution for θ_2. That means $2\lambda/d = 1$, for an angle of $\theta_2 = 90°$. This requires a minimum of $d = 2\lambda = 6.0\,\text{cm}$.

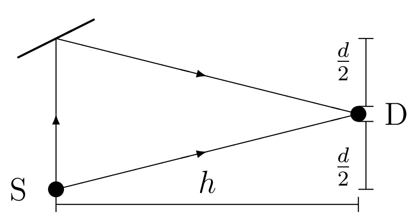
We don't want to see $m = \pm 3$, and for $\sin\theta_3 = 3\lambda/d$ not to have a solution, the maximum possible $d = 3\lambda = 9.0\,\text{cm}$.

Now, a peak just at 90° would not really be visible in the lab. So pick a d value somewhere in between: say $d = (9+6)/2 = 7.5\,\text{cm}$. Then, the angle for $m = 2$ will be $\theta_2 = \sin^{-1}(6/7.5) = 53°$, which should be easily observable.

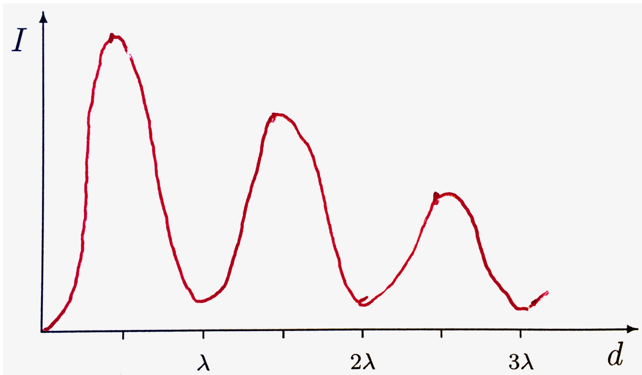
(b) If you had a diffraction grating instead of two slits, the intensity peaks would have been sharper, making a more precise experiment. Why do you think I didn't give you a grating? (For example, do you think its cost might have been too high?)

Answer: A proper grating requires lots—thousands at least—of slits. With a slit spacing around 5.5 cm, the diffraction grating required would be over 50 m long—hardly the kind of thing to fit on a tabletop in our lab.

6. (30 points) You have the following interferometer setup in a lab, with a wave source labeled S, detector D, and a mirror to produce two wave paths between source and detector. There is a distance of d between the source and the mirror, and you keep the length h constant while changing d. The wavelength of the waves is λ.



Make a qualitative graph of how the intensity detected will depend on the distance d. Be careful about the relative peak heights, and whether the intensity is ever exactly zero. Explain your decisions, and how you made your graph.



Answer: There are two paths for waves between the source and detector. You can see from the geometry of the experiment that the path length difference is $l_1 - l_2 = d$. There is also a phase shift of $\lambda/2$ due to the reflection off the mirror. Therefore, the condition for constructive interference is $d = (m + \frac{1}{2})\lambda$, while destructive interference happens at $d = m\lambda$, where m is an integer.

The intensity of the waves will decrease with distance from the source. Therefore, the peak heights will become smaller as d increases. And since the wave amplitudes will, for the same reason, not match exactly, there will be no full cancellation and no $I = 0$ except for $d = 0$ when the source and mirror are practically on top of each other.