Assignment 8; Phys 186 Name ________________

1. (30 points) A physics major tells you that they have a new explanation of black holes. They ask you to imagine a brick, which we heat up to higher and higher temperatures.

- Atoms are made of electrically charged particles.
- As temperature increases, the atoms making up the brick will jiggle more vigorously.
- More vigorously jiggling atoms will radiate higher amplitude electromagnetic waves.
- More electromagnetic waves in the brick will lead to increased destructive interference between the waves;
- Therefore the intensity of the radiation escaping from the brick will decrease.
- Since mass is equivalent to energy, heating up the brick while less and less radiation escapes is equivalent to adding more and more mass.
- Beyond a threshold, *no* radiation will escape: the brick will be completely black. We can calculate the threshold using $E = mc^2$.

This reasoning is incorrect. Circle the bullet points that seem wrong or misleading to you and briefly explain why they make the reasoning incorrect. Make calculations where needed: for example, compare the energy needed to raise the brick's temperature by $1\,\mathrm{K}$ (revisit your first semester physics) to mc^2.

2. (30 points) For the following, you will need the expression for the radius of the event horizon of a black hole that we derived.

(a) One way to get a black hole is to squeeze lots of mass into a very small volume. Almost all the mass of a Hydrogen atom is squeezed into the very small volume of a proton. Are protons dense enough to be black holes? Answer this question by using reasonable numbers about protons you can look up. Write down the sources of your numbers.

(b) You stand such that your lower legs, with mass 5.0 kg, is at the event horizon of a 30 solar mass black hole, and your 5.0 kg head is just 1.0 m further away from the horizon. Find the difference between the gravitational forces felt by your head and your lower legs. What effect on you would this difference in forces have? (Math hint: When $d \ll r$, $\frac{1}{r^2} - \frac{1}{(r+d)^2} \approx \frac{2d}{r^3}$.)

3. (40 points) If you incorporate quantum effects, it turns out that black holes have a temperature. And as with any object that has a temperature, black holes therefore emit thermal radiation. If you look up the black hole surface temperature, you will find

$$T = \frac{hc^3}{2^4\pi^2 G k_B M}$$

Where h is Planck's constant, c is the speed of light, G is the universal gravitational constant, k_B is the Boltzmann constant, and M is the mass of the black hole. You will remember, from your first semester of physics, that the rate of heat loss by electromagnetic radiation from an object at temperature T is $dQ/dt \propto T^4$ (you may have seen this as $Q/\Delta t \propto T^4$; it's the same thing). Go look up that equation, and

(a) Find the equation for the *rate of mass loss* of a black hole due to thermal radiation. Simplify the expression as much as you can.

(b) Calculate the rate of mass loss of a supermassive black hole at the center of a galaxy with $M = 10^9\, M_\odot$, where $M_\odot$ stands for a solar mass. You should find that such a large black hole is not in danger of evaporating—explain why your result means this.

(c) Find the mass of a microscopic black hole where the rate of mass loss is equal to its own mass per second. If you could somehow compress an everyday object into a black hole, would it last?

Extra Problems (not graded)

4. (0 points) One estimate for the average mass density of our universe is the mass of one H atom per 3 cubic meters.

(a) What is the radius of a black hole that has that mass density? Compare your result to the size of our *observable* universe (look it up).

(b) A friend of yours now suggests that it is impossible that we live inside a black hole. After all, everything that falls into a black hole gets ripped apart and gets compressed into an incredibly high density. But we're alive! What do you think of that argument? Explain.

5. (0 points) You stand such that your lower legs, with mass 5.0 kg, is at the event horizon of a black hole with mass M, and your 5.0 kg head is just 1.0 m further away from the event horizon.

(a) Find the difference ΔF between the gravitational forces felt by your head and your lower legs, for black holes with mass $M = 10^{18}$ kg, 10^{21} kg, 10^{24} kg, 10^{27} kg, 10^{30} kg, 10^{33} kg, 10^{36} kg, and 10^{39} kg.

When $d \ll r$, $\left(\frac{1}{r^2} - \frac{1}{(r+d)^2}\right) \approx \frac{2d}{r^3}$. When $d \gg r$, $\left(\frac{1}{r^2} - \frac{1}{(r+d)^2}\right) \approx \frac{1}{r^2}$. If, however, r and d are comparable in magnitude, you will have to calculate ΔF without using either of these approximations.

(b) Make a graph of $\log_{10} \Delta F$ on the vertical axis and $\log_{10} M$ on the horizontal axis. Connect the dots. On the graph, also indicate: the mass of the Earth, the mass of the sun, the mass of a 30 solar mass black hole, and the mass of a thousand solar mass black hole. Then, also indicate where the tidal stretching will begin to be dangerous for humans: if a tension force of 1000 N were applied to us, we could only withstand that with great pain after just minutes.

Note that it's important in science to communicate your results effectively. I expect you to make good choices and a clear presentation with this graph.

If a spaceship were to approach the event horizon of a supermassive black hole with a billion solar masses, would they have to worry about tidal forces?