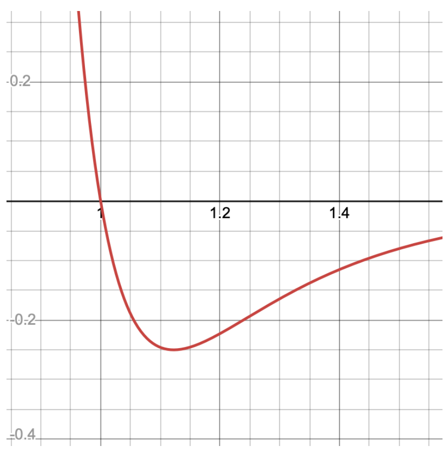
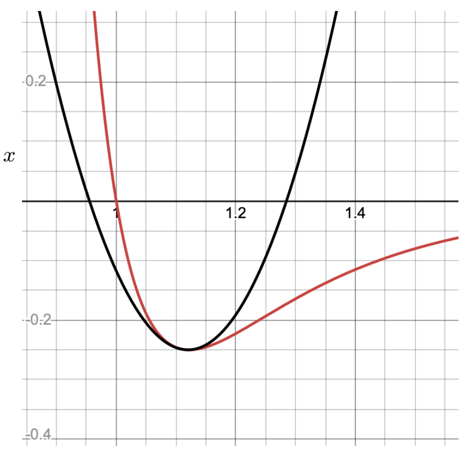
# Solutions to Exam 1; Phys 186

**1. (25 points)** You have a mass $m$ that can move in one dimension ($x$) under the influence of a force that has a potential energy $U(x)$ (and no other forces). The first graph below is a graph with $x$ on the horizontal axis and $U(x)$ on the vertical. On the right, as $x \to \infty$, $U(x) \to 0$, but on the left, as $x$ becomes smaller, $U(x)$ grows without limit.




(a) You let go of the mass, at rest, from an an initial position $x_0 > 0$. Describe the conditions under which the mass will oscillate back and forth.

**Answer:** The total energy $K + U(x)$ will be constant. Notice that if the total energy is positive, the mass can go off to in infinity on the right hand side, and its kinetic energy will always be greater than zero. That's not an oscillation. You start from rest, so with zero kinetic energy. So the only way to have $K + U > 0$ is if the initial $x$ is far to the left: $x < 1$ on the graph.

With $x > 1$, $K + U < 0$, so the mass will go back and forth between two positions where $K = 0$. ($K = \frac{1}{2}mv^2$ can't be negative!). In other words, it will oscillate.

(b) In the second graph, you see a parabola fitted to be as close as possible to $U(x)$ when $x$ is very close to $x_{\text{min}}$, the equilibrium point where $U$

is a minimum; at equilibrium, the fit is exact. The equation for this parabola is $y(x) = a(x - x_{\text{min}})^2 - b$. What is $\omega$, the angular frequency for oscillations with very small amplitudes, where $x$ is close to $x_{\text{min}}$? Explain your reasoning. *Hint:* Not all of $a$, $b$, or $x_{\text{min}}$ are relevant to oscillations.

**Answer:** $b$ and $x_{\text{min}}$ are irrelevant—they just shift where the energy minimum is located on the graph; they don't change the shape of the parabola. The $a$ in the parabola equation plays the role of $\frac{1}{2}k$ in the spring potential energy $U_s = \frac{1}{2}kx^2$. Since for a spring $\omega = \sqrt{k/m}$, in this case

$$\omega = \sqrt{\frac{2a}{m}}$$

*Note:* Chemical bond potential energies often look like this. You can get vibrations in molecules, as long as the energy is not large enough to break the bond. For very small oscillations, molecular vibrations are exactly like springs.

**2. (25 points)** The diffraction gratings you looked through had a line (slit) density of 500 lines/mm stamped on them. But manufacturers' numbers, you have found, are not always exact. The best way to determine your line density is to use a reference light source that produces a known wavelength $\lambda_{\text{ref}}$. (This would be the light equivalent of a tuning fork for sound; the best such sources use dilute gases of a single chemical element under a high voltage. After all, atoms are the same everywhere.) You find a source with $\lambda_{\text{ref}} = 612\,\text{nm}$, shine it though your diffraction grating and observe the $m = 1$ peak corresponding to that same color of light at an angle of 16.95°. What, therefore, was the line density of your diffraction grating?

**Answer:** Since $\sin\theta_m = m\lambda/d$,

$$d = \frac{\lambda_{\text{ref}}}{\sin\theta_{\text{ref}}} = 2099\,\text{nm}$$

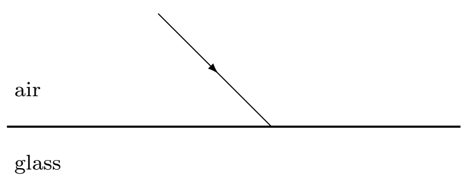
The line density then will be

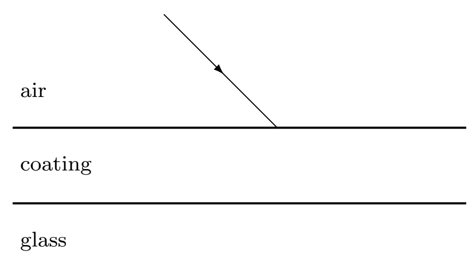
$$\frac{1}{d} = 4.76 \times 10^{-4}\,\text{lines/nm} = 476\,\text{lines/mm}$$

That's close to the manufacturer's value, as it should be.

**3. (25 points)** Say you need corrective lenses for your vision, and you have some glasses made. But after you receive your glasses, you wonder if you should spend a little extra and get a layer of antireflective coating applied to your lenses. This coating will be very thin, but will have a constant thickness. And since you have taken physics, you realize that the extra coating will have an index of refraction of its own, distinct from air and glass. Light incident on the coating will refract at an angle different than glass, and since it is angles that matter, even a very thin coating will make a difference. So you wonder if the coating will affect the focal length of your lens, making it useless to correct your vision.

Consider the following two situations: light going from air into glass, and light going through the extra layer of coating before making it into the glass. Produce an argument, including relevant calculations, that tells you whether you need to worry about applying the coating. You won't need numerical values for anything.




**Answer:** With light going from air into glass, we have

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}}$$

Since the interfaces between air and the coating and glass and the coating are parallel to one another (constant thickness coating), the entry angle of light into the coating and the exit angle of light from the coatinbg are the same; call that $\theta_{\text{coat}}$. Therefore we have

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{coat}} \sin\theta_{\text{coat}} \quad \text{and} \quad n_{\text{coat}} \sin\theta_{\text{coat}} = n_{\text{glass}} \sin\theta_{\text{glass}}$$

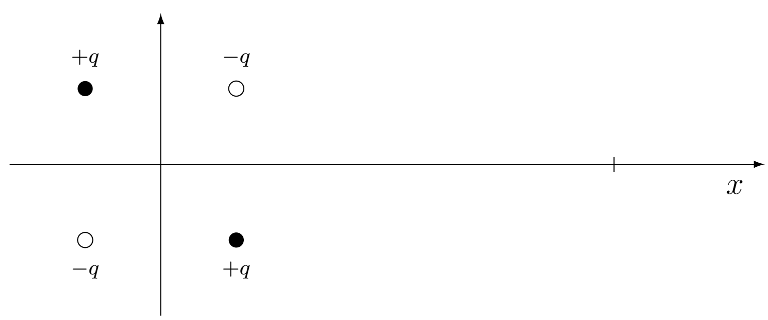
Combining these equations, we get the same result as for air into glass:

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}}$$

In other words, the coating does not affect the angle into the glass, as long as it has constant thickness. Therefore it won't affect the focal length of your lens, so you don't have to worry about anything.

**4. (25 points)** Four charges are arranged on a square with sides $a$, and the origin of the axes is at the center of the square.

(a) Find the $x$- and $y$-components for the total electric field at an arbitrary point on the $x$-axis, where $x > a/2$. Also draw an arrow depicting the total electric field vector at this point. *Hint:* Notice that this arrangement is the same as two dipoles side by side, and just use the dipole result from your notes to avoid almost all calculations.




**Answer:** This is a combination of two dipoles, exactly like what we solved in class, except that one of them is upside down and they are displaced by $\pm a/2$ along the $x$-axis. Therefore, we use the single dipole result, $E_x = 0$ and $E_y = -kqa[x^2 + (a/2)^2]^{-3/2}$.

For the dipole on the right, we take the negative of the results, since the + and − charges are reversed, and replace $x$ with $x - a/2$. For the dipole on the left, we replace $x$ with $x + a/2$, and add the components. $E_x = 0$ and

$$E_y = kqa\left[\frac{1}{\left((x-\frac{a}{2})^2+(\frac{a}{2})^2\right)^{3/2}} - \frac{1}{\left((x+\frac{a}{2})^2+(\frac{a}{2})^2\right)^{3/2}}\right]$$

There's no need to simplify.

(b) When $x \gg a$, the electric field magnitude behaves like $E \propto 1/x^n$. What is $n$? *Math hint:* $[(x+\alpha)^2+\beta^2]^{-3/2} \approx x^{-3}(1-3\alpha/x)$ when $|\alpha| \ll x$ and $|\beta| \ll x$. *General hint:* This is worth only 5% of the exam. If you're uncomfortable with algebra, leave this until the very end.

**Answer:** We can use the math hint with the expression above, with $\alpha = -a/2$ and then $\alpha = a/2$:

$$E_y \approx \frac{kqa}{x^3}\left[\left(1+\frac{3a}{2x}\right)-\left(1-\frac{3a}{2x}\right)\right] = \frac{3kqa^2}{x^4}$$

Therefore $n = 4$. Note that for a bare charge, $n = 2$, and for a dipole, $n = 3$. This electric field is even weaker than a dipole; it's known as a *quadrupole.*