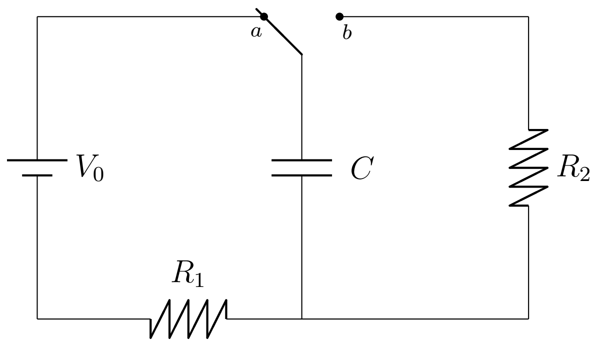
Solutions to Assignment 5; Phys 186

1. (20 points) Here is a simplified (oversimplified) model of a circuit for a camera flash. The resistance R_1 is considerably larger than R_2. When the switch is at a, the capacitor C slowly recharges. When the switch is at b, C rapidly discharges.



(a) Say the switch remains at a for a long time in order to fully charge up the capacitor. This is a "long time" compared to what?

Answer: The time scale for charging up is R_1C—so the time must be long compared to R_1C.

(b) What is the power dissipated by R_2 immediately after the switch is flipped to b? Explain, using this, why a flash requires a small value for R_2.

Answer: Since the capacitor was fully charged, the voltage across it immediately after the switch is flipped will be V_0. (It would not have had any time to discharge yet.) Therefore, using a loop equation, the voltage across the resistor will also be V_0 and the current going through will be V_0/R_2. The power is then

$$P = \frac{V_0^2}{R_2}$$

A flash requires a large burst of energy delivered in a short amount of time. Therefore P should be large—which is why R_2 should be small.

(c) Say $C = 12\ \mu$F, and $R_2 = 0.21\ \Omega$. How long will it take for the capacitor to discharge 90% of its starting charge?

Answer: The capacitor needs to go down to $1 - 0.9 = 01$ of its original charge. Uisng the exponential discharge relationship,

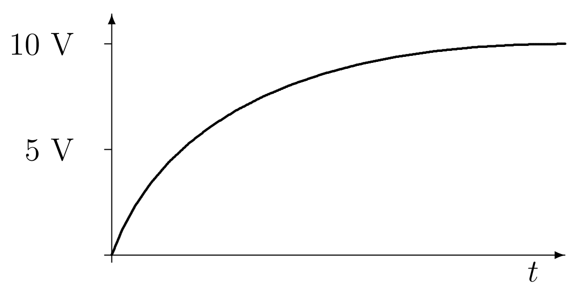
$$Q = Q_0 e^{-t/R_2 C} \qquad \Rightarrow \qquad \frac{Q}{Q_0} = 0.1 = e^{-t/R_2 C}$$

Therefore

$$t = -R_2 C \ln 0.1 = 5.8 \times 10^{-6}\ \text{s}$$

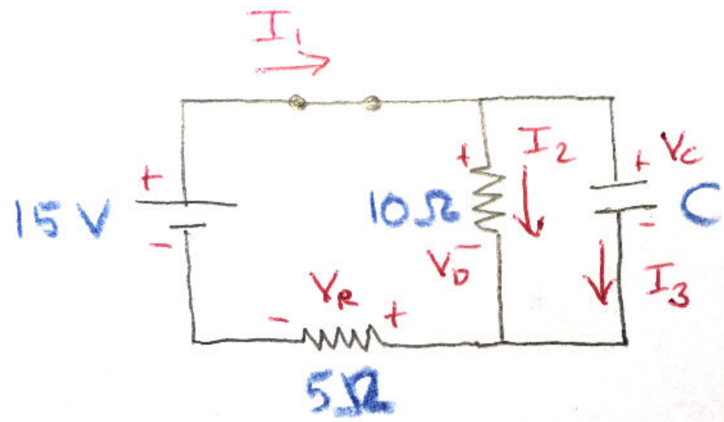
2. (30 points) You have a capacitor (its capacitance is not important), a switch, wires, a 15.0 V DC battery, a 5.0 Ω resistor, and a device that behaves like a 10.0 Ω resistor.

(a) You want the voltage across your device to behave like the following graph after you close the switch; starting at 0.0 V and gradually going up to 10.0 V:



Draw a circuit diagram for the circuit that will do this. Write the junction and loop equations and show that immediately after you close the switch and a long time after you close the switch, the voltage across your device will be 0.0 V and 10.0 V.

Answer: Notice that the voltage graph looks exactly like that for a capacitor charging up. So you should connect your device in parallel with the capacitor, forcing them to have the same voltage. Circuit:

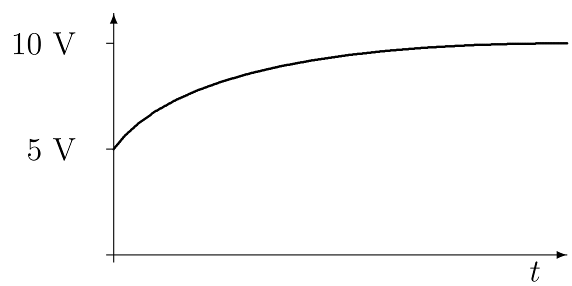


Junction: $I_1 = I_2 + I_3$. Loops: $15\,V = V_D + V_R$ and $V_D = V_C$.

At $t = 0$, the capacitor still has no charge, so $V_C = 0$. Therefore $V_D = 0$ as well, which is what we want.

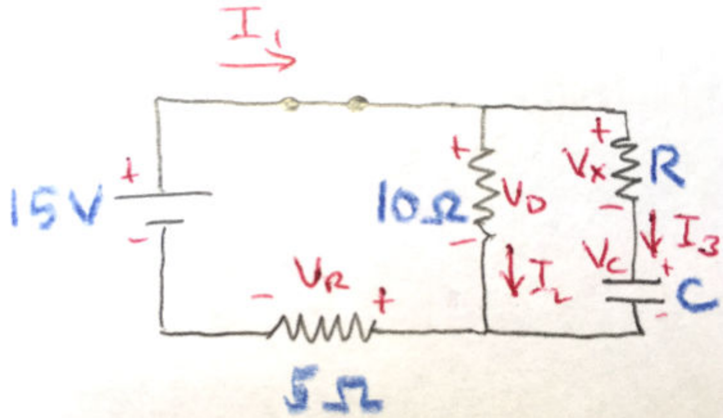
At large times, the capacitor will have fully charged up, so no current will go through it. Therefore $I_3 = 0$. Therefore $I_1 = I_2$ and $15\,V = (10\,\Omega)I_1 + (5\,\Omega)I_1$, which means $I_1 = 1$ A and $V_D = (10\,\Omega)I_1 = 10$ V.

(b) Let's say that instead of the situation in (a), your device requires a voltage graph looking like the following, starting at 5.0 V and gradually going up to 10.0 V:



You can accomplish this by adding an extra resistor R to the circuit that you had for (a). Draw the circuit with the extra resistor R, and use loop and junction equations to calculate the value of R for which the voltage across the device will be 5.0 V immediately after closing the switch and 10.0 V a long time after.

Answer: Circuit:



Junction: $I_1 = I_2 + I_3$. Loops: $15\,V = V_D + V_R$ and $V_D = V_C + V_x$.

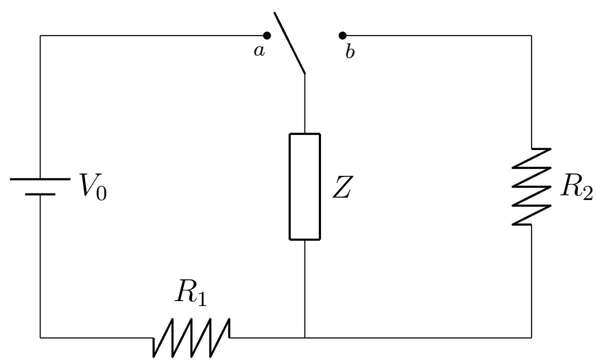
At $t = 0$, $V_C = 0$ again, plus we know that $V_D = 5$ V. Therefore $V_x = 5$ V and $V_R = 10$ V. In that case, $I_1 = V_R/(5\,\Omega) = 2$ A, $I_2 = V_D/(10\,\Omega) = 0.5$ A. Using the junction equation, $I_3 = 2 - 0.5 = 1.5$ A. So $R = V_x/I_3 = 5/1.5 = 3.33\,\Omega$.

At large times, no current goes through the capacitor, which is exactly the same situation as in part (a), so as before, $V_D = 10$ V.

3. (50 points) A capacitor stores energy in the electric field between its plates. It takes time for the field to change, so the voltage across a capacitor can't change instantaneously. But the current through a capacitor can change instantaneously, for example when you open or close a switch. All this means that if you have a capacitor with zero electric field, its voltage at that instant has to be zero, but its current can be anything. But when the electric field is at its maximum, the voltage across the capacitor will have the appropriate value, but the current now will have to be zero.

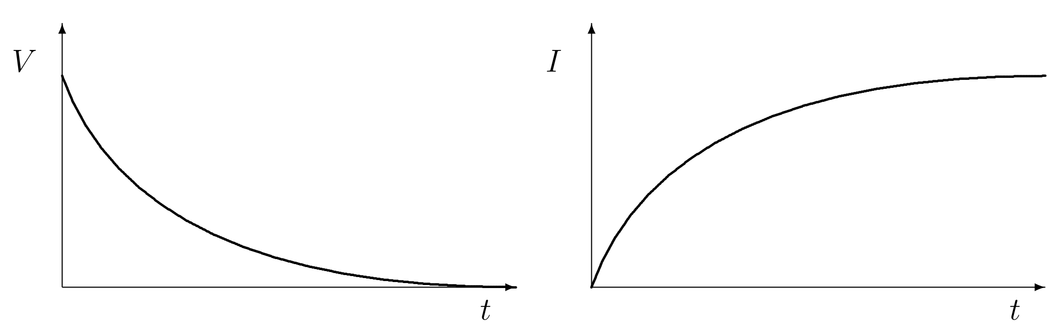
I now give you a device I will call a zinglschritt. A zinglschritt stores energy in the magnetic field in its coils. It takes time for the field to change, so the current through a zinglschritt can't change instantaneously. But the voltage across a zinglschritt can change instantaneously, for example when you open or close a switch. All this means that if you have a zinglschritt with zero magnetic field, its current at that instant has to be zero, but its voltage can be anything. But when the magnetic field is at its maximum, the current through the zinglschritt will have the appropriate value, but the voltage now will have to be zero.

You then have the following circuit that builds up or brings down the magnetic field in a zinglschritt, depending on whether the switch is at position a or b. The rectangle represents your zinglschritt.



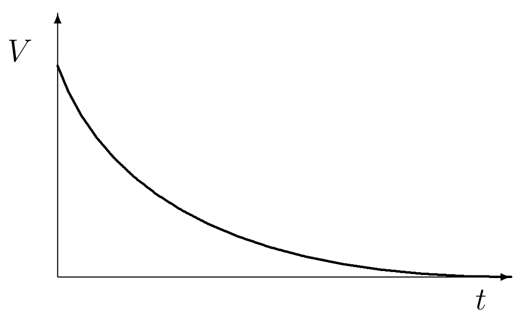
(a) You start with no magnetic field in the zinglschritt, with the switch neither connected to a nor b. Then you connect it to a. Immediately after the switch is set to a, what is the current through the zinglschritt, and the voltage across the zinglschritt? You then wait a long time, so that the magnetic field reaches its maximum value. What, then, is the current through the zinglschritt, and the voltage across the zinglschritt? Make rough sketches of how the voltage and current depend on time, with $t = 0$ as the time you set the switch to a. Explain your reasoning, or provide calculations.

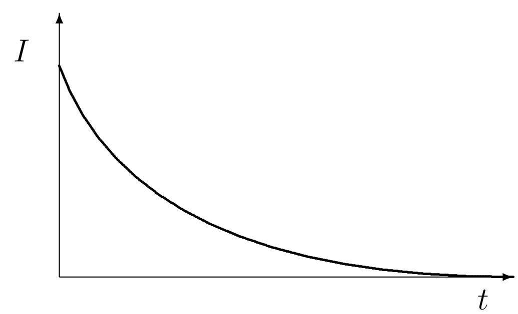
Answer: The loop equation when the switch is at a is $V_0 = V_z + R_1 I$. At $t = 0$, the current $I = 0$. This means that $V_z = V_0$ at the beginning. After a long time passes, $V_z = 0$, so the current becomes $I = V_0/R_1$. The curves should be such that the voltage and current approach their long time values gradually, not abruptly.



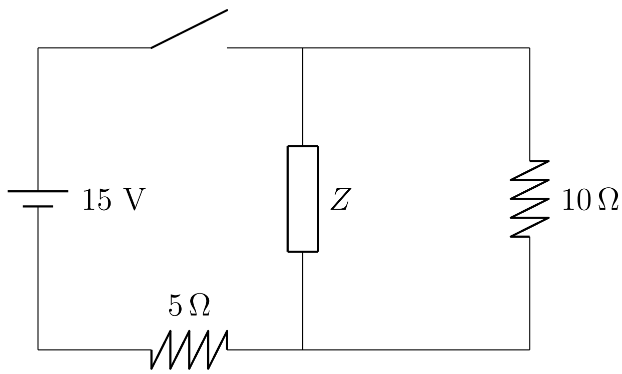
(b) After having built up the magnetic field in the zinglschritt with the switch at a for a long time, you then flip the switch to b. Immediately after the switch is set to b, what is the current through the zinglschritt, and the voltage across the zinglschritt? You then wait a long time. What, then, is the current through the zinglschritt, and the voltage across the zinglschritt? Make rough sketches of how the voltage and current depend on time, with $t = 0$ as the time you set the switch to b. Explain your reasoning, or provide calculations.

Answer: The loop equation with the switch is at b is $V_z = R_2 I$. When $t = 0$, the current $I = V_0/R_1$, since the current through a zinglshritt cannot change rapidly. This means that $V_z = R_2 V_0/R_1$ at the beginning. After a long time passes, $V_z = 0$, so the current also becomes $I = 0$.





(c) You now have the following circuit, which includes a zinglschritt. The switch has been open for a long time. You then close the switch at time $t = 0$. Calculate the voltage across the $10\,\Omega$ resistance at time $t = 0$, when the switch has just closed, and then the voltage after a long time has passed and the magnetic field in the zinglschritt has reached its maximum value. Then sketch a graph of how the voltage across the $10\,\Omega$ resistance depends on time.

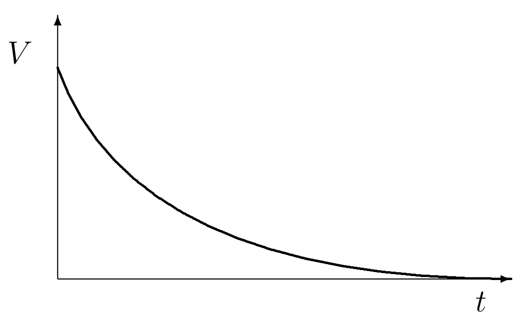


Answer: The junction equation: $I_1 = I_2 + I_3$, where I_1 is the current through the battery and the $5\,\Omega$ resistance, I_2 is the current through the zinglschritt, and I_3 is the current through the $10\,\Omega$ resistance.

The loop equations with $V = RI$ for the resistors: $15\,\mathrm{V} = V_z + (5\,\Omega)I_1$, and $V_z = (10\,\Omega)I_3$.

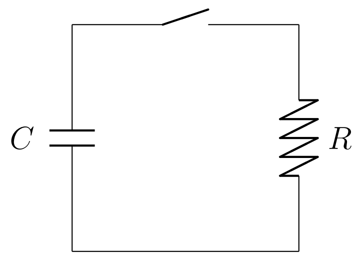
At $t = 0$, we have $I_2 = 0$. Therefore, $I_1 = I_3$. Putting this into the loop equations, we get $15\,\mathrm{V} = (10\,\Omega)I_1 + (5\,\Omega)I_1$, giving $I_1 = 1\,\mathrm{A}$. This means that the voltage across our device is $(10\,\Omega)(1\,\mathrm{A}) = 10\,\mathrm{V}$.

At long times, we have $V_z = 0$. Therefore the device voltage is also 0.



Extra Problems (not graded)

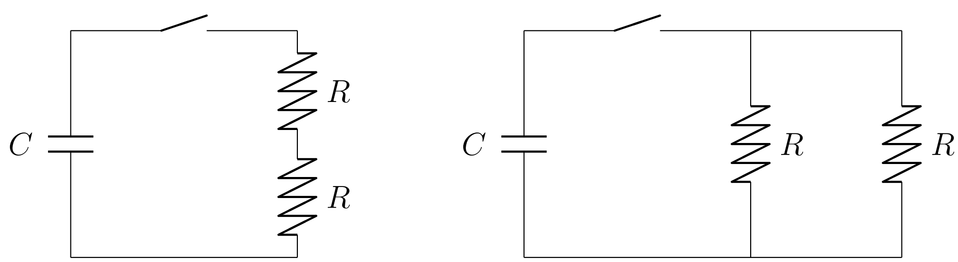
4. (0 points) In the classroom, we discussed the following circuit for discharging a capacitor. The capacitor is fully charged at first, with voltage V_0 across it, when the switch is open; when the switch is closed at time $t = 0$, the charge starts to decline. We also discussed that since the current in the circuit after the switch is closed is due to the charges on the capacitor plates moving away, that the larger the current, the faster the capacitor will be discharged. Since a larger resistance R means a smaller current, the capacitor will take longer to discharge, with the current through the capacitor $I_C(t) = I_C(0)\, e^{-t/RC}$ and $I_C(0) = V_0/R$.



Now say you have two identical resistances, both R, which you hook up (1) in series, and (2) in parallel, and you replace your original R with these two two-resistor configurations.

(a) Draw a diagram for each circuit, (1) and (2). Write down junction and loop equations for each circuit.

Answer:



In the series case, there are no junctions and a single current I_C, and the loop equations gives $V_C = 2RI_C$.

In the parallel case, we have $I_C = I_1 + I_2$ and the loop equations $V_C = V_1$ and $V_1 = V_2$. Putting these together, $RI_1 = RI_2 = V_C$, and therefore $I_1 = I_2$ and $I_C = 2I_1$.

(b) Find expressions for $I_{C1}(0)$ and $I_{C2}(0)$, the currents through the capacitor at time $t = 0$ for each circuit, in terms of variables such as V_0, R, and C. Which circuit, therefore, will discharge faster?

Answer: Using the loop and junction equations, for the series case, at $t = 0$ we start with $V_C = V_0 = 2RI_C$, therefore $I_C(0) = V_0/2R$. For the parallel case, $I_C(0) = 2V_0/R$.

Since the current is higher for the parallel case, it will discharge faster.

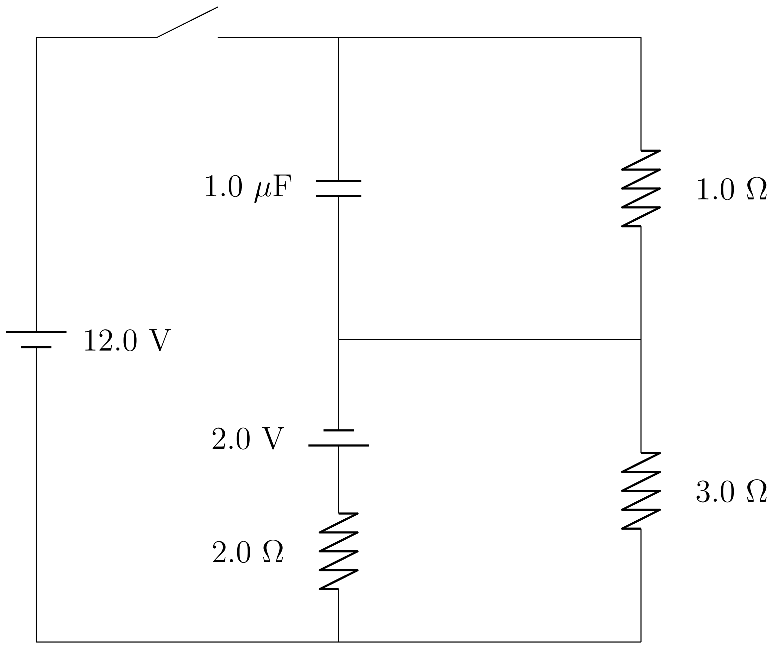
(c) Compare your result in (b) to the single-R circuit. All that has changed is the effective resistance values that appear in $I_C(0)$; after all, C is the same. So, by analogy, write down expressions for $I_{C1}(t)$ and $I_{C2}(t)$, for all times t. (Ask me for help if you need it!) Qualitatively sketch a graph of these two currents versus time, representing both currents on the same graph.

Answer: When comparing to the single resistor case, where $I_C(0) = V_0/R$, we can see that the effective resistance for the series case is $2R$, while for the parallel case it is $R/2$. The times scales for the exponential discharge, then, will be $2RC$ and $RC/2$, with expressions

$$I_{C1}(t) = \frac{V_0}{2R}\, e^{-t/2RC} \qquad \text{and} \qquad I_{C2}(t) = \frac{2V_0}{R}\, e^{-2t/RC}$$

When you graph these, the exponential decay of $I_{C2}(t)$ will start from an initial value that is four times that of $I_{C1}(t)$, but it will also fall four times as fast. Those of you who remember a bit of calculus might note that the area under both of these curves is the total charge of the capacitor, and hence the same.

5. (0 points) Just before the switch is closed, the capacitor in the following circuit is completely discharged. You then close the switch.



(a) Find the currents through all the resistors *immediately after* the switch is closed, before the capacitor has any time to charge up at all.

Answer: If the capacitor has had no time to build up any charge, the voltage across it must be $V = Q/C = 0$. The 1 Ω resistor is in parallel with the capacitor, hence it sees the same voltage of 0. Therefore no current flows through this resistor—all the current will go through the capacitor, bypassing this resistor. $I_1 = 0$. In other words, an uncharged capacitor behaves like a closed switch, shorting out anything in parallel to it.

This simplifies the circuit: in effect, we can replace the capacitor with a straight wire, and remove the 1 Ω resistor from the circuit. We now analyze the circuit with the two remaining resistors. The junction equation is

$$I_0 = I_2 + I_3$$

where I_0 is the current put out by the 12 V battery. The two loop equations are

$$12 \text{ V} + 2 \text{ V} = (2\ \Omega)I_2$$

$$(2\ \Omega)I_2 = 2\text{ V} + (3\ \Omega)I_3$$

The first loop equation directly gives $I_2 = (12+2)/2 = 7$ A. Putting that into the second loop equation produces $I_3 = (2 \cdot 7 - 2)/3 = 4$ A.

(b) Find the currents through all the resistors *a very long time after* the switch is closed, after the capacitor has completely charged up.

Answer: After a long time, the capacitor will have fully charged, and the current through it will be zero. All the current will go through the parallel 1 Ω resistor. In other words, in this case we remove the capacitor, replacing it with an open switch. The circuit equations are very similar, with $I_0 = I_1$ now going through the 1 Ω resistor as well.

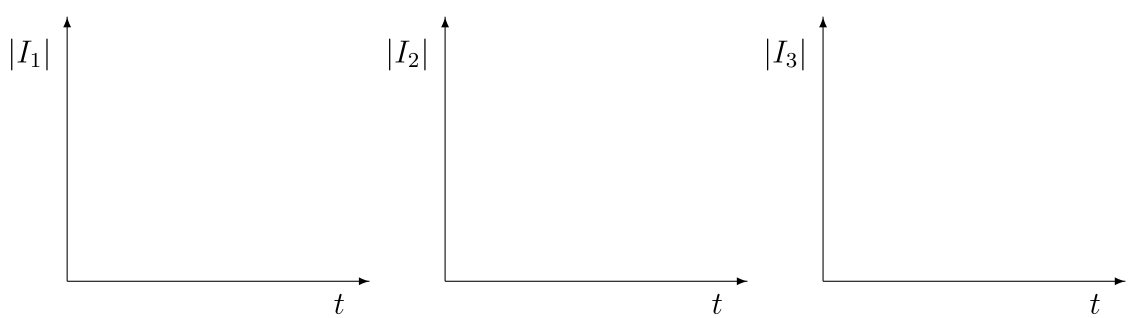
$$I_1 = I_2 + I_3$$

$$12\text{ V} + 2\text{ V} = (2\ \Omega)I_2 + (1\ \Omega)I_1$$

$$(2\ \Omega)I_2 = 2\text{ V} + (3\ \Omega)I_3$$

Solving, we end up with $I_2 = 4$ A, $I_3 = 2$ A, and $I_1 = 6$ A.

(c) Sketch qualitative I vs t graphs for each of the currents. The switch closes at $t = 0$.



Answer: I_1 will rise from 0 and level off asymptotically at 6 A. I_2 and I_3 will have exponential decay curves between their $t = 0$ and $t \to \infty$ values.