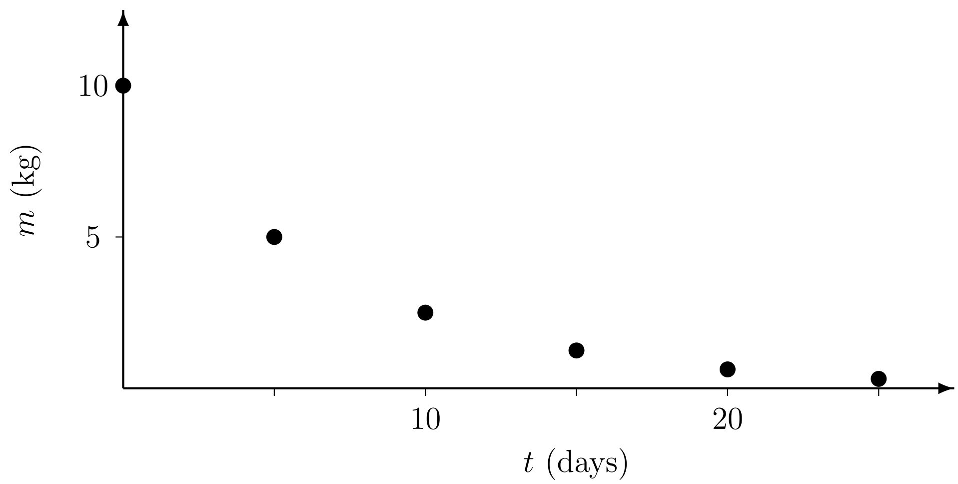
# Solutions to Assignment 9; Phys 186

**1. (30 points)** Let's do half-lives.

(a) You have friend who is not a science major. She tells you that quantum mechanical events cannot be truly random. After all, randomness implies unpredictability, but physicists make precise predictions using quantum mechanics. Given the half-life, they can tell you exactly what amount of a radioactive sample will remain after a certain time. Given the energy of photons emitted from a light source, they can calculate the interference pattern observed when a diffraction grating is placed between the source and a screen. Correct your friends' misconceptions and explain what the role of randomness in quantum mechanics is.

**Answer:** *Individual* quantum events are random. We cannot predict when an individual nucleus will decay, or the exact path any single photon will take. But precisely *because* individual events are random, if lots and lots of individual random events take place, a very reliable statistical pattern will emerge. We get very definite half-lives and interference patterns when the number of events is about in the $10^{20}$'s.

(b) You have a 10.0 kg block of radioactive material $A$, and at time $t = 0$, you start with all 10 kg being pure $A$. Sketch a graph of the amount of $A$ that remains in the block over time. The half-life of $A$ nuclei is 5.0 days. **Answer:**




(And connect the dots.)

(c) Pick, from among the following, the correct expression for the amount of $A$ remaining over time. Here $\tau = t_{1/2}/\ln 2$, and $m_0 = 10$ kg.

(a) $m = m_0 \cos \frac{t}{\tau}$

(b) $m = m_0 \left(1 - \frac{t}{\tau}\right)$

(c) $m = m_0 \ln \frac{t}{\tau}$

**(d)** $m = m_0\, e^{-\frac{t}{\tau}}$

(e) $m = \frac{m_0}{\sqrt{1-(\frac{t}{\tau})^2}}$

(d) What is $m$ at $t = 9.0$ days?

**Answer:** $m = 10\, e^{-(9 \ln 2)/5} = 2.9$ kg

**2. (20 points)** There have been occasional concerns that people living close to electric power lines might have high risks of cancer due to their proximity to electromagnetic radiation emitted by the power lines. Determine, using a simple calculation, whether this concern is warranted. I will supply any physical data that you need—just ask me. I'm interested in whether you know to ask the right questions about the numbers that you might need.

**Answer:** Power lines carry alternating current, with a frequency of 60 Hz. That means accelerating electric charges with a 60 Hz frequency, producing predominantly 60 Hz electromagnetic waves. Photons with this frequency have an energy of

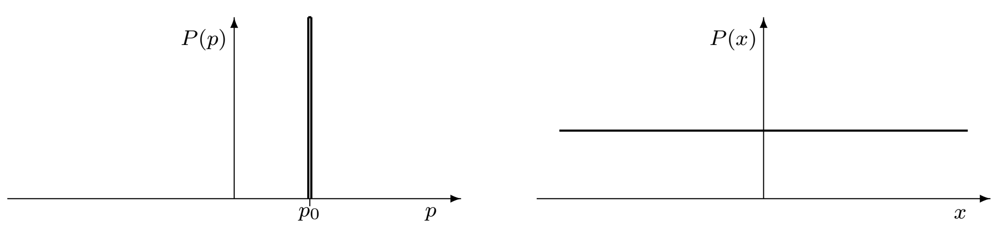
$$E = hf = 4.0 \times 10^{-32} \text{ J} = 2.5 \times 10^{-13} \text{ eV}$$

Chemical bond energies are of the order of eV's. So the photons produced by power lines have energies that are extremely smaller than bond energies. Therefore they cannot disrupt biologically important molecules such as DNA, and therefore it is hard to see any physical mechanism for their causing cancer.

**3. (30 points)** Recall the experiment we did in the lab, where we produced an electron beam and bent its trajectory by using electric and magnetic fields.

(a) Say you measure the momentum of one of the electrons, finding that it is exactly $p = p_0$. Now, if you really have learned the exact momentum of an electron, this must mean that if you instantaneously re-measure the momentum of the electron, not giving it any time to change after the first measurement, you must be *guaranteed* to get the same result $p = p_0$. Therefore, immediately after the measurement, you should know the probability distributions $P(p)$ and $P(x)$ associated with that electron. Sketch these probability distributions:

**Answer:** Being guaranteed to obtain $p = p_0$ means that the probability for all $p \neq p_0$ must be zero. Indeed, since we are certain about the momentum, the standard deviation $\Delta p = 0$. The probability distribution $P(p)$must be a zero-width spike. And from the uncertainty principle, this means that $\Delta x = \infty$: we know nothing about $x$. This is indicated by a totally flat probability distribution that makes no distinction between $x$ values.




(b) In your lab experiment, say you accelerated the electrons, starting from rest, by applying a voltage of 6000 V. Find $v$, the speed of the electrons after their acceleration is complete, in two ways: by using the *nonrelativistic* and then *relativistic* expressions for kinetic energy. Was it necessary to account for relativity?

**Answer:** Setting the initial electric potential energy equal to the final kinetic energy,

$$eV_a = \tfrac{1}{2}m_e v^2 \quad \Rightarrow \quad v = \sqrt{\frac{2eV_a}{m_e}} = 4.59 \times 10^7 \text{ m/s} = 0.153c$$

The relativistic calculation:

$$eV_a = (\gamma - 1)m_e c^2 \quad \Rightarrow \quad \gamma = 1.0117$$

But $\gamma$ depends on $v$, so that

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = 0.152 \quad \Rightarrow \quad v = 4.56 \times 10^7 \text{ m/s}$$

The answers are almost the same. The error introduced by using the nonrelativistic kinetic energy is very small; indeed, much smaller than all the other experimental errors in this lab. (As you saw with your very inaccurate results for $e/m_e$.)

(c) There is a distance $L$ between the point the electrons traveling at $v$ emerge from the electron gun and when they hit the screen. If $L = 0.10$ m, circle what you think is a good estimate for $\Delta x$, the uncertainty in the position of the electrons along the direction in which they are traveling. (*Note:* $\hbar = h/2\pi$.)

$$\underline{L} \qquad \frac{\hbar L}{2} \qquad \frac{\hbar}{2L} \qquad \frac{c}{L} \qquad \frac{\gamma c}{L}$$

Now write down your estimate for the uncertainty: $\Delta x = \underline{0.10}$ m.

(d) The electrons are subatomic particles: you have to use quantum mechanics. Since $\Delta x < \infty$, this means that $\Delta p > 0$. In other words, you cannot assume that you know that the electrons are traveling exactly at the speed $v$ you calculated. Estimate $\Delta p$, assuming that $\Delta p$, your uncertainty about $p$, is at its *minimum* possible value.

**Answer:** For the minimum,

$$\Delta p = \frac{\hbar}{2\Delta x} = 5.28 \times 10^{-34}\,\mathrm{kg \cdot m/s}$$

(e) Physicists usually like their electron beams to be "monochromatic"—all particles at a single wavelength. If the value of $\Delta p/p$ is small, the beam is close to monochromatic. Calculate the numerical value of $p$ for your beam of electrons, and then find $\Delta p/p$ and decide whether this beam is almost monochromatic or not.

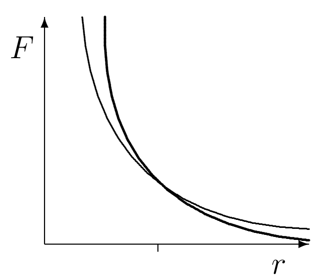
**Answer:** The momentum is $p = mv = 4.15 \times 10^{-23}\,\mathrm{kg{\cdot}m/s}$. Therefore

$$\frac{\Delta p}{p} = 1.27 \times 10^{-11} \ll 1$$

Since this number is so small, the beam is nearly monochromatic.

**4. (20 points)** The following graphs show $F$, the magnitude of force, on the vertical axis, and $r$, the distance between two protons, on the horizontal axis. The tick mark on the horizontal $r$-axis indicates an approximate separation of $10^{-15}$ m. The curve that is the same in in all graphs describes how the electrical force between the protons depends on $r$. The curve that is different represents proposals for how the strong nuclear force between the protons depends on $r$.

(c)




Select the graph that shows the correct qualitative behavior for the strong nuclear force. And then explain—give the reasons for your choice.

**Answer:** Protons in an atomic nucleus are separated by about $10^{-15}$ m. That means that at such $r$ and smaller, the strong nuclear force must overcome the electrical repulsion—it must have a larger magnitude. But at larger than nuclear distances, we notice no effects of the strong force. So at large distances, it must be considerably weaker than electrical forces. Graph (c) is the only one that shows this.

# Extra Problems (not graded)

**5. (0 points)** Consider a particle with mass $m$ confined to a 1D box, so that it is impossible to find the particle outside $0 \leq x \leq L$. Other than the confinement, the particle is not interacting with anything, so its energy is purely kinetic energy. Now say that you have a particle in the lowest energy level, the ground state, with energy $E_1 = h^2/8mL^2$. This means that you can calculate the particle's momentum:

$$\frac{h^2}{8mL^2} = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{p^2}{2m}$$

Solving for $p$, we get

$$\sqrt{p^2} = \sqrt{\frac{h^2}{4L^2}} \qquad \Rightarrow \qquad p = \frac{h}{2L}$$

But notice that since $h$ and $L$ are known exactly, therefore $p$ is known exactly. And this means there is no uncertainty in your knowledge of the momentum: $\Delta p = 0$. But since the particle is confined, $\Delta x \approx L$; in any case, $\Delta x < \infty$. Therefore

$$\Delta x \Delta p = 0 < \frac{h}{4\pi}$$

The Uncertainty Principle is violated! But this can't be right. Find the error in the reasoning above. Some options for you to consider:

- Maybe $\Delta x = \infty$ because the particle can quantum tunnel outside the box.
- Maybe $h$ is not known exactly, so its uncertainty needs to be taken into account.

- Maybe $\Delta p > 0$ because there is a subtle mistake in the calculation.

*Hint:* Remember that momentum is a vector!

**Answer:** There *is* a mistake in the calculation.

Physically, if the momentum was $p = h/L$, it would mean the particle would be continually moving in the $+x$ direction. But it can't—it hits the wall of the box. The argument overlooks the fact that a square root has two values:

$$\sqrt{\frac{h^2}{L^2}} = \pm\frac{h}{L}$$

In other words, the particle may also be moving in the $-x$ direction. In fact, the probabilities of both directions are equal. Since it is not certain whether the particle has $p = +h/L$ or $p = -h/L$, the uncertainty $\Delta p > 0$.

**6. (0 points)** You want to measure the half-life of a radioactive sample. You first measure the background radiation in your lab: 23 counts per minute. Then, every day at 12:00 noon, you take a Geiger counter and count the total number of events with your sample in place for exactly one minute. Call this activity $A_T$. Here is a table of your data:

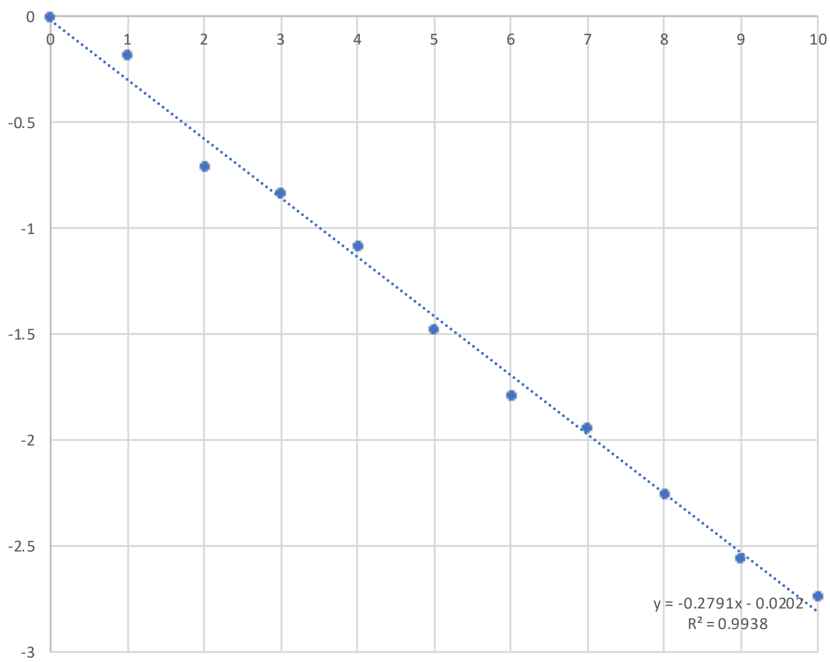
| $t$ (days) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_T$ (counts/minute) | 123 | 111 | 84 | 79 | 70 | 59 | 52 | 49 | 44 | 40 | 38 |

(a) Make a graph involving $A_S$, *the activity due to your sample alone*, and time. Choose two of the following to put on the axes of your graph:

$$t \qquad 2^{-t} \qquad \log_2 t \qquad t^{-2} \qquad \frac{A_S}{A_{S0}} \qquad 2^{-A_S/A_{S0}} \qquad \log_2\left(\frac{A_S}{A_{S0}}\right) \qquad \left(\frac{A_S}{A_{S0}}\right)^{-2}$$

$A_{S0}$ refers to $A_S$ at day 0. (Math reminder: $2^x = e^{x\ln 2}$ and $\log_2 x = \ln x/\ln 2$.)

**Answer:** Use $t$ on the horizontal axis and $\log_2\left(\frac{A_S}{A_{S0}}\right)$ on the vertical.




(b) Using the slope of a straight line through your data, calculate the half-life.

**Answer:** Since

$$A_S = A_{S0}\, 2^{-t/t_{1/2}} \qquad \Rightarrow \qquad \log_2\left(\frac{A_S}{A_{S0}}\right) = \left(-\frac{1}{t_{1/2}}\right) t$$

Therefore

$$t_{1/2} = -\frac{1}{\text{slope}} = 3.6 \text{ days}$$