Solutions to Exam 4; Phys 186

1. (10 points) You are doing the double slit part of your microwave optics lab, where you measure the intensity of the microwave beam after passing it through two slits. You notice that when you measure the intensity straight down the middle ($\theta = 0$), the intensity is *more than double* the intensity you observe when you block one of the slits.

(a) Briefly explain why.

Answer: Down the middle, the waves from each slit have an equal distance to travel, so you will have peak meeting peak, trough meeting trough. That doubles the amplitude, and since the intensity $I \propto A^2$, that will result in a more than doubling of the intensity.

(b) Intensity is related to energy. But then, if the intensity more than doubles with both slits unblocked, wouldn't you be violating energy conservation? Explain.

Answer: The *total* energy is still conserved. At angles $\theta \neq 0$, you also have destructive interference and everything in between. Extra intensity in some spots is compensated for by less intensity in others.

2. (25 points) You have photons produced by electrons in H dropping down from energy level E_3 to E_1, with $E_n = -|E_1|/n^2$, where $n = 1, 2, 3, \ldots$ and $|E_1| = 13.6\,\text{eV}$. You then pass these photons through a diffraction grating with a line density of 100 lines/mm, and measure θ_1, the angle at which you see the $m = 1$ maximum for these photons.

(a) The source of your photons is in the lab. What would θ_1 be? Your diffraction grating will probably not be suitable for measuring this wavelength. Why? What sort of diffraction grating would you seek to replace yours with, if you wanted to measure the wavelength properly?

Answer: Energy conservation means the energy of the emitted photon is equal to the difference between energy levels:

$$E = hf = E_3 - E_1 = (-13.6\,\text{eV})\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 12.1\,\text{eV}$$

The wavelength:

$$\lambda_s = \frac{hc}{E} = 1.03 \times 10^{-7}\,\text{m}$$

The angle for this UV light is, for a grating with $d = 1/100 \times 10^3\,\text{m}$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda_s}{d}\right) = \sin^{-1}\left[(1.03 \times 10^{-7}\,\text{m})(100 \times 10^3\,1/\text{m})\right] = 0.6°$$

That's a very small angle, and it may be difficult to resolve properly for accurate measurements. To increase the angle, you should a try a diffraction grating with a higher line density (smaller d).

(b) The source of your photons is a galaxy that is billions of light years away. You now measure $\theta_1 = 20.0°$. What is v/c, where v is the speed at which the galaxy is receding away from us? *Hint:* You should use the result of one of your assignment problems.

Answer: Your observed, highly redshifted wavelength is

$$\lambda_o = d \sin\theta_1 = \frac{\sin 20°}{100 \times 10^3\,1/\text{m}} = 3.42 \times 10^{-6}\,\text{m}$$

Use, from assignment 7,

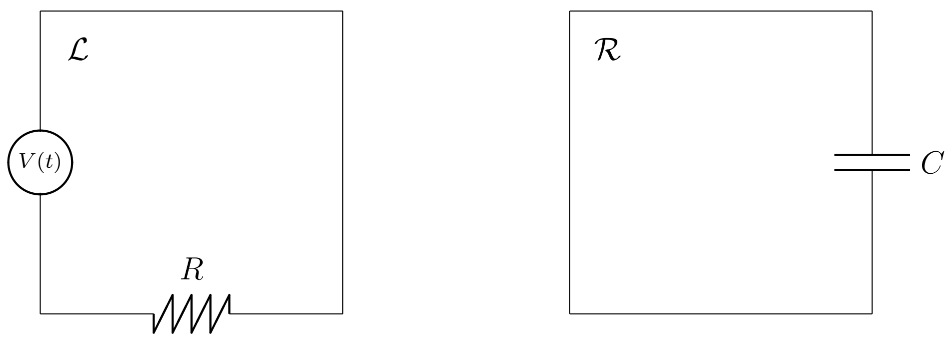
$$\frac{\lambda_o}{\lambda_s} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

Solving for v/c, we get

$$\frac{v}{c} = \frac{\left(\frac{\lambda_o}{\lambda_s}\right)^2 - 1}{\left(\frac{\lambda_o}{\lambda_s}\right)^2 + 1} = 0.998$$

Extremely far away galaxies will be receding from us close to the speed of light, and their light will be extremely redshifted.

3. (35 points) You have two circuits next to each other:



Circuit $\mathcal{L}$ has a voltage source which is a function generator that produces a sinusoidal waveform $V(t) = V_0 \cos(2\pi f t)$, with $f = 250\,\text{Hz}$.

Math: $\frac{d}{dt}\cos\omega t = -\omega \sin\omega t$; $\frac{d}{dt}\sin\omega t = \omega\cos\omega t$.

(a) Sketch a graph of the voltage across R vs time. (This is trivial.)

Answer: This is just your basic $V(t) = V_0\cos(2\pi f t)$ graph. The period is $T = 1/f = 4\,\text{ms}$.

(b) Sketch a graph of the voltage induced in circuit $\mathcal{R}$. (Don't worry about the exact amplitude: just get the shape right. Also don't worry about the minus sign in electromagnetic induction.) Explain how you arrived at your conclusion.

Answer: Use $V_{\text{ind}} = -\frac{d}{dt}\Phi_B$. The magnetic flux through circuit $\mathcal{R}$ is $\Phi_B \propto B \propto I \propto V(t)$. Therefore

$$V_{\text{ind}} = -\frac{d}{dt}\Phi_B \propto \frac{d}{dt}V(t)$$

You can derive the rate of change from the slopes of your previous graph. The result should be

$$V_{\text{ind}} \propto f \sin(2\pi f t)$$

(The f is for future reference; it doesn't have to be in your answer.)

(c) Now sketch a graph of the waveform you will see if you measure the current in circuit $\mathcal{R}$ under these conditions. Explain how you arrived at your conclusion. *Hint:* We did something very similar in class, plus you played with a very similar circuit in your Lab Exam.

Answer: We did the AC power connected to a capacitor in class; alternatively, you played with it in your Lab Exam. The capacitor charge is $Q = CV_{\text{ind}}$, and $I_C = \frac{d}{dt}Q = C\frac{d}{dt}V$ Therefore

$$I_C \propto \frac{d}{dt}V \propto f^2 \cos(2\pi f t)$$

(Again, the f^2 is for later.)

(d) You now double the frequency on your power source to 500 Hz, without changing the amplitude. Sketch a graph of the waveform you will see if you measure the current in circuit $\mathcal{R}$. Explain how you arrived at your conclusion.

Answer: Looking at $I_C \propto f^2 \cos(2\pi f t)$, doubling the frequency will produce a graph with a quadrupled amplitude and a halved period of 2 ms. If you just follow the graphical procedure of tracing the slopes, you will still notice the increased amplitude.

4. (30 points) You have a particle under the influence of a force that acts exactly like a spring force with constant k. Let x stand for the distance from equilibrium, and p the momentum of the particle. The particle speed $v \ll c$.

(a) Write down the total energy of the particle as a function of x and p, with m and k as constants.

Answer: The nonrelativistic kinetic energy plus the potential energy is

$$E_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2m}p^2 + \tfrac{1}{2}kx^2$$

where $p = mv$ is the nonrelativistic momentum.

(b) If this was a classical (non-quantum) particle, would its minimum possible total energy be zero or non-zero? Explain.

Answer: The minimum energy is zero, which corresponds to a motionless ($p = 0$) particle at the equilibrium point ($x = 0$).

(c) If this was a quantum particle, would its minimum possible total energy be zero or non-zero? Explain.

Answer: If the particle had $p = 0$ and $x = 0$, we would be completely certain about its momentum and location: $\Delta p = 0$ and $\Delta x = 0$. This would then mean that $\Delta x \Delta p = 0 < h/4\pi$, which violates the uncertainty principle.

The particle will instead have a probability distribution for its location and momentum, with non-zero standard deviations. And since it will not be stationary and at equilibrium, its total minimum energy will be larger than zero.