Homework Solutions 5 (Griffiths Chapter 5)

12 At an azimuthal angle θ on the sphere, the ring of charge has a radius of $R\sin\theta$ and produces a magnetic field of

$$d\mathbf{B} = dI\,\frac{\mu_0}{2}\,\frac{R^2\sin^2\theta}{\left(R^2\sin^2\theta + R^2\cos^2\theta\right)^{3/2}}\,\hat{\mathbf{z}} = dI\,\frac{\mu_0\sin^2\theta}{2R}\,\hat{\mathbf{z}}$$

The current dI is due to a charge $Q(Rd\theta)(2\pi R\sin\theta)/4\pi R^2$. This charge takes a time of $2\pi/\omega$ to rotate full-circle. Therefore the current is

$$dI = \frac{Q(Rd\theta)(2\pi R\sin\theta)}{4\pi R^2}\,\frac{\omega}{2\pi} = d\theta\,\frac{Q\omega\sin\theta}{4\pi}$$

Integrating,

$$\mathbf{B} = \frac{\mu_0 Q\omega}{8\pi R}\,\hat{\mathbf{z}}\int_0^\pi d\theta\sin^3\theta = \frac{\mu_0 Q\omega}{6\pi R}\,\hat{\mathbf{z}}$$

32 The vector potential is

$$d\mathbf{A} = \frac{\mu_0 r'^4\omega\rho\,dr'}{3}\,\frac{\sin\theta}{r^2}\,\hat{\boldsymbol{\phi}}\quad\text{for } r'<r,\quad d\mathbf{A} = \frac{\mu_0 r'\omega\rho\,dr'}{3}\,r\sin\theta\,\hat{\boldsymbol{\phi}}\quad\text{for } r'>r$$

Integrating,

$$\mathbf{A} = \frac{\mu_0\omega\rho\sin\theta}{3}\,\hat{\boldsymbol{\phi}}\left(\frac{1}{r^2}\int_0^r dr'\,r'^4 + r\int_r^R dr'r'\right) = \frac{\mu_0\omega\rho\sin\theta}{3}\,\hat{\boldsymbol{\phi}}\left(\frac{rR^2}{2} - \frac{3r^3}{10}\right)$$

Then,

$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A} = \frac{\mu_0\omega\rho}{3}\left[\cos\theta\left(R^2 - \frac{3}{5}r^2\right)\hat{\mathbf{r}} - \sin\theta\left(R^2 - \frac{6}{5}r^2\right)\hat{\boldsymbol{\theta}}\right]$$

39 As in (5.12), the current due to a spinning uniform charge q is $I = q\omega/2\pi$.

(a) For each infinitesimally wide circle,

$$d\mathbf{m} = dI\,\pi r^2\,\mathbf{z}\quad\text{with}\quad dI = \sigma(2\pi r)dr\,\frac{\omega}{2\pi} = dr\,\sigma r\omega$$

Therefore

$$\mathbf{m} = \pi\sigma\omega\,\hat{\mathbf{z}}\int_0^R dr\,r^3 = \frac{\pi\sigma\omega R^4}{4}\,\hat{\mathbf{z}}$$

(b) Treat this like (5.12), where dI is the same:

$$d\mathbf{m} = \left(d\theta \, \frac{Q\omega \sin\theta}{4\pi} \right) \pi (R \sin\theta)^2 \, \hat{\mathbf{z}}$$

Therefore,

$$\mathbf{m} = \frac{Q\omega R^2}{4} \, \hat{\mathbf{z}} \int_0^\pi d\theta \, \sin^3\theta = \frac{Q\omega R^2}{3} \, \hat{\mathbf{z}}$$

This results in a dipole vector potential

$$\mathbf{A}_{\text{dip}} = \frac{\mu_0 Q\omega R^2}{12\pi r^2} \, \hat{\mathbf{z}} \times \hat{\mathbf{r}} = \frac{\mu_0 Q\omega R^2 \sin\theta}{12\pi r^2} \, \hat{\boldsymbol{\phi}}$$

Since $Q = 4\pi R^2 \sigma$, this is also

$$\mathbf{A}_{\text{dip}} = \frac{\mu_0 \sigma\omega R^4 \sin\theta}{3r^2} \, \hat{\boldsymbol{\phi}}$$

which is the same as equation (5.69) for $r \geq R$.

44 Hall effect:

(a) Right-hand rule: the + charges will be deflected down.

(b) The electric and magnetic force magnitudes must be equal, so $qvB = qE$. The voltage is $V = Et$; therefore the Hall voltage $V = vBt$.

(c) If the charge-carriers flipped sign, the charges will be deflected in the opposite direction, and the Hall voltage will change sign.

61 Gyromagnetic ratio:

(a) With $I = Q\omega/2\pi$, $m = I\pi r^2 = Q\omega r^2/2$. The angular momentum $L = Mr^2\omega$. Therefore

$$\frac{m}{L} = \frac{Q\omega r^2}{2M\omega r^2} = \frac{Q}{2M}$$

(b) Each infinitesimal ring will have the same gyromagnetic ratio; therefore, nothing will change: $m/L = Q/2M$.

(c) For a classical electron,

$$m = \frac{Q}{2M}\frac{\hbar}{2} = 4.61 \times 10^{-24} \mathrm{A} \cdot \mathrm{m}^2$$