Homework Solutions 6 (Griffiths Chapter 6)

8 The bound currents:

$$\mathbf{J}_b = \boldsymbol{\nabla} \times \mathbf{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(sks^2\right)\hat{\mathbf{z}} = 3ks\,\hat{\mathbf{z}}$$

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = ks^2\hat{\boldsymbol{\phi}} \times \hat{\mathbf{s}}\Big|_{s=R} = -kR^2\hat{\mathbf{z}}$$

This means a z-independent, cylindrically symmetric current distribution, so Ampère's Law should work the same way as it does with an infinite current-carrying wire. Drawing a circular loop around the z-axis, for $s < R$,

$$B(2\pi s) = \mu_0\, 2\pi \int_0^s ds'\, s'(3ks') = 2\pi\mu_0 ks^3 \quad \Rightarrow \quad \mathbf{B} = \mu_0 ks^2\,\hat{\boldsymbol{\phi}}$$

For $s > R$, the current through the loop includes the surface current:

$$2\pi\mu_0 kR^3 + \mu_0(2\pi R)(-kR^2) = 0 \quad \Rightarrow \quad \mathbf{B} = 0$$

16 Using the version of Ampère's Law with $\mathbf{H}$ and a loop between the tubes

$$\oint d\mathbf{l} \cdot \mathbf{H} = (2\pi s)H = I_f \quad \Rightarrow \quad \mathbf{H} = \frac{I}{2\pi s}\,\hat{\boldsymbol{\phi}}$$

$$\mathbf{B} = \mu_0(1+\chi_m)\mathbf{H} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\,\hat{\boldsymbol{\phi}}$$

With $\mathbf{M} = \chi_m\mathbf{H}$, the bound current density is

$$\mathbf{J}_b = \boldsymbol{\nabla} \times \mathbf{M} = 0$$

On the inner tube, the bound surface current is

$$\mathbf{K}_b = \mathbf{M} \times (-\hat{\mathbf{s}}) = \chi_m\frac{I}{2\pi a}\,\hat{\mathbf{z}}$$

On the outer tube,

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{s}} = -\chi_m\frac{I}{2\pi b}\,\hat{\mathbf{z}}$$

Now, with the total current through the loop,

$$\oint d\mathbf{l} \cdot \mathbf{B} = (2\pi s)B = \mu_0\left[I + 2\pi a\left(\chi_m\frac{I}{2\pi a}\right)\right] \quad \Rightarrow \quad \mathbf{B} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\,\hat{\boldsymbol{\phi}}$$