1. (80 points) Consider the simple harmonic oscillator with an added drag force,

$$F_{d,x} = -bv_x = -b\frac{dx}{dt}$$

(This is the simplest type of drag, called "viscous drag." This system is called the "damped oscillator.")

Try an exponential solution for the resulting differential equation.

There are two important cases and one edge case: κ has two complex solutions with non-zero imaginary parts; κ has two real solutions (zero imaginary parts); and κ has only a single real solution. The edge case divides the two important cases. Write the conditions for satisfying each scenario.

When κ has two real solutions, what does x(t) look like? Write an equation and sketch a graph.

When κ has two complex solutions, what does x(t) look like? Write an equation and sketch a graph.

2. (20 points) Consider the ordinary (undamped) simple harmonic oscillator, with motion

$$x(t) = A\cos(\omega t + \delta)$$

Explain the physical meanings of A, ω , and δ , and how they relate to the motion of the oscillator.

Derive (don't just state) equations for the frequency f and period T of the oscillator. *Hint:* Use the periodicity of the cosine function.