

A *sinusoidal traveling wave* has the form

$$y(x, t) = A \cos(kx - \omega t - \delta)$$

Notice its similarity to the harmonic oscillator.

1. **(20 points)** This is a traveling wave, so it should equal  $F(x - vt)$  for some  $F$  and  $v$ . What is the speed of this wave (in terms of  $A$ ,  $k$ ,  $\omega$ ,  $\delta$ )?
2. **(20 points)** As in your last assignment, interpret the physical meaning of  $A$ ,  $k$ ,  $\omega$ , and  $\delta$ , and find relations between these variables and the period  $T$  and wavelength  $\lambda$  of the wave.
3. **(60 points)** Waves can travel in both directions along a string simultaneously. Suppose we have two sinusoidal traveling waves:

$$y(x, t) = A \cos(kx - \omega t) + B \cos(-kx - \omega t)$$

These waves have the same frequency but different directions and possibly different amplitudes.

Now suppose we clamp our string at two points:  $x = 0$  and  $x = L$ . This means that  $y = 0$  at those two points at *all* times  $t$ . Use that condition to eliminate variables: you should be able to find  $B$  in terms of  $A$ , and  $k$  in terms of  $L$ . You should note that there are multiple solutions for  $k$ —find the condition for allowed  $k$  values. A helpful trigonometric identity:  $\cos(A + B) = \cos A \cos B - \sin A \sin B$ .

You should find that only special values of  $k$  (and therefore  $\omega$ ) are allowed, but the amplitude is not restricted. These waves are called *standing waves* (because they have no overall translational motion) and the special frequencies are *resonant* or *harmonic* frequencies.