

When you include time-dependence, quantum wave functions are solutions to the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

The *Hamiltonian* operator $\hat{H}$ is the total energy. With a 1D particle in a box, that's just the kinetic energy: $\hat{H} = \hat{K}$. All quantum states can be expressed as a superposition of the energy eigenfunctions we have found before. Say your wave function combines the first and second energy levels:

$$\Psi(x, t) = N [\psi_1(x)e^{-i\omega_1 t} + i\psi_2(x)e^{-i\omega_2 t}]$$

Here, ψ_n refers to the n th energy eigenfunction and $\omega_n = E_n/\hbar$. N is a normalization constant.

1. **(30 points)** Show that Ψ satisfies the Schrödinger equation. (Plug it in.)
2. **(20 points)** You know that the ψ_n are normalized:

$$\int_0^L dx \psi_1^* \psi_1 = \int_0^L dx \psi_2^* \psi_2 = 1$$

Show that ψ_1 and ψ_2 are *orthogonal*:

$$\int_0^L dx \psi_1^* \psi_2 = 0$$

3. **(50 points)** Find N , the normalization constant.