

## PHYS 191 Activity 5: Gauss's Law

1. Sketch electric field line diagrams for three highly symmetric charge distributions: the point  $q$ , the infinite line with uniform charge density  $\lambda$ , and the infinite plane with uniform charge density  $\sigma$ . Make your best attempt at a 3D diagram for each. Show multiple perspectives if necessary.

Remember the rules for field lines: they start (end) on positive (negative) charges, they never cross, and the strength of the field is proportional to the number of lines per unit area.

Next to each diagram, write the proportionality of the electric field to distance  $r$  (for example, for the point charge  $q$ ,  $E \propto 1/r^2$ .)

**Answer:** Wrap a sphere around the point charge. Rotational and reflection symmetry dictates that the field lines will be of uniform density and be directed radially outward. The number of lines going through concentric spheres of different radii will remain constant, but the area the lines go through will increase like  $4\pi r^2$ , the surface area of a sphere. So the field strength, proportional to the areal density of lines, will go like  $E \propto 1/r^2$ .

Wrap a cylinder of length  $l$  and radius  $r$  around the line of charge. Rotational and reflection symmetry dictates that the field lines will be of uniform density and be directed radially outward. Therefore no lines will go *through* the end caps of the cylinder. The number of lines going through the sides of concentric cylinders of different radii will remain constant, but the area the lines go through will increase like  $2\pi r l$ , the surface area of the side of a cylinder. So the field strength, proportional to the areal density of lines, will go like  $E \propto 1/r$ .

Draw a cylinder (or any pillbox shape) with end caps on either side of the plane, parallel to the plane. Rotational and reflection symmetry dictates that the field lines will be parallel, of uniform density and be directed outward from the plane. No lines will go through the sides of the pillbox. The number of lines going through the caps of pillboxes with different lengths will remain constant, and the area will also be constant. So the electric field has to be uniform, not depending on  $r$ .

2. With field lines,  $E \propto \frac{\text{number of lines crossing a surface}}{\text{unit area}}$ , and number of lines  $\propto$  charge. How could you use these proportionalities to determine how much charge is inside a region, by only using the electric field line diagram far away from the charges?

**Answer:** Count the lines crossing a closed surface enclosing the charges.

3. The mathematical measure of the number of field lines crossing a surface is the *electric flux*,  $\Phi_E = \int \vec{E} \cdot d\vec{a}$ . Here,  $d\vec{a} = da \hat{n}$ , with  $da$  an infinitesimal area and  $\hat{n}$  the unit vector perpendicular to a surface. The dot product picks out the part of the electric field perpendicular to the surface, i.e. only the field lines that cross the surface, not skim along its surface. The integral sums up over the whole surface under consideration.

Find the electric flux of the point charge field through a sphere of radius  $R$  centered

at the charge, and the electric flux of the infinite line charge field through a cylinder of radius  $R$  and length  $L$  centered along the line.

**Answer:** First, the point. The unit vector  $\hat{\mathbf{n}}$  is directed radially outward, and so is the electric field;  $\vec{E} \parallel \hat{\mathbf{n}}$ . Therefore  $\vec{E} \cdot d\vec{a} = E da$ .

$$\Phi_E = \int_{\text{sphere}} \vec{E} \cdot d\vec{a} = E \int_{\text{sphere}} da = E (4\pi R^2)$$

Now the line. On the end caps of the cylinder,  $\vec{E} \perp \hat{\mathbf{n}} \perp$ , and so  $\vec{E} \cdot d\vec{a} = 0$ . On the sides,  $\vec{E} \parallel \hat{\mathbf{n}}$  and  $\vec{E} \cdot d\vec{a} = E da$ .

$$\Phi_E = \int_{\text{cyl side}} \vec{E} \cdot d\vec{a} = E \int_{\text{cyl side}} da = E (2\pi RL)$$

4. Gauss's Law:

$$\Phi_E = \frac{Q_{\text{in}}}{\epsilon_0}$$

Use Gauss's Law to determine the electric field of a hollow shell with uniform charge  $Q$  and radius  $a$ , both inside and outside the shell. You will have to (i) decide the direction of the field using symmetry; (ii) pick a good surface to integrate over; (iii) calculate the enclosed charge as a function of the size of the surface.

**Answer:** The direction of the field, from symmetry, will be radial. The surfaces to integrate over need to respect the rotational symmetry: they will be spheres. The argument with the point charge applies:  $\Phi_E = 4\pi R^2 E$ .

First, inside the sphere:  $R < a$ . The total charge inside the Gaussian surface will be zero. Therefore  $E = 0$ .

Outside the sphere,  $R > a$ ,  $Q_{\text{in}} = Q$ .

$$\Phi_E = 4\pi R^2 E = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad E = \frac{Q}{4\pi\epsilon_0 R^2}$$