Solutions 5; Phys 191 Name ______________

1. (60 points) Use Gauss's Law to determine the electric field of a solid, uniformly charged sphere (total charge Q, radius R) at all points both inside and outside the sphere. Make sure to explain how you (1) use symmetry (2) determine flux and (3) find enclosed charge.

Answer: Wed have spherical rotational symmetry. The electric field direction will therefore be radial. For $r > R$, the charge enclosed in a spherical Gaussian surface will be Q, and $\Phi_E = E\,(4\pi r^2)$. Therefore, outside the sphere,

$$E = \frac{Q}{4\pi\epsilon_0\, r^2}$$

Inside, $Q_{\rm in} \neq Q$. For $r < R$,

$$Q_{\rm in} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q = \frac{r^3}{R^3} Q$$

So, for $r < R$,

$$E = \frac{Q_{\rm in}}{4\pi\epsilon_0\, r^2} = \frac{Q\, r}{4\pi\epsilon_0\, R^3}$$

2. (40 points) In class, we did part of the work to determine the electric field produced by an infinite line with uniform charge density λ, and the infinite plane with uniform charge density σ. Using Gauss's Law, finish the job for each.

Answer: The Gaussian surface around the line of charge is a cylinder with length l and radius r.

$$\Phi_E = E\,(2\pi r l) = \frac{Q_{\rm in}}{\epsilon_0} \quad \Rightarrow \quad E = \frac{Q_{\rm in}}{l}\frac{1}{2\pi\epsilon_0\, r} = \frac{\lambda}{2\pi\epsilon_0\, r}$$

The Gaussian surface for the plane is the pillbox with endcap area A. The electric flux goes entirely through the endcaps, so

$$\Phi_E = E(2A) = \frac{Q_{\rm in}}{\epsilon_0} \quad \Rightarrow \quad E = \frac{Q_{\rm in}}{A}\frac{1}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}$$