Solutions 6; Phys 191 Name ________________

1. (100 points) You have a ring with radius R and charge Q distributed uniformly. Calculate the electric potential due to the ring along its axis of symmetry (the z-axis). You will have to figure out the right integral to do, by analogy with the electric field:

$$\begin{aligned} \vec{E} &= k \int dq \, \frac{\hat{\mathbf{r}}}{r^2} \\ V &= \int dq \, (???) \end{aligned}$$

Then set up the integral geometry (very similar to integrating to find the field as we did in Activity 4) and evaluate the integral.

Finally, calculate E_z by taking the appropriate derivative of V. Your answer should reproduce what we found in class by doing the electric field integral.

Answer: For a point charge, we found $U = kqQ/r$. So the electric potential for a point charge is $V = kq/r$.

For an infinitesimal dq along the ring (the same dq as in Activity 4), $dV = k(dq)/r$ with $dq = Q\frac{d\phi}{2\pi}$ and $r = \sqrt{R^2 + z^2}$.

$$V = k \int_{\text{ring}} dq r = k \frac{Q}{2\pi r} \int_0^{2\pi} d\phi = k \frac{Q}{r} = k \frac{Q}{\left(R^2 + z^2\right)^{1/2}}$$

Given this,

$$E_z = -\frac{\partial V}{\partial z} = -kQ\left(-\frac{1}{2}\right)\left(R^2 + z^2\right)^{-3/2} 2z = \frac{kQz}{(z^2 + R^2)^{3/2}}$$

as in Activity 4.