Solutions to Assignment 6; PHYS 185

1. (30 points) You have a glass with mass m bouncing off the floor. Its duration of contact with the floor—the time difference between first touching the floor and last touching the floor—is τ. Its velocity when it first touches the floor is v_{iy}, and its velocity as it launches off the floor is v_{fy}. If, at any time during its contact with the floor, the force on the glass ever exceeds F_{max}, even for the tiniest fraction of a second, the glass will break. Note that the force on the glass during its collision with the floor will *not* be constant, and you have no idea what the exact force graph looks like.

(a) Write down an inequality describing the condition under which it is certain that the glass *will* break. *Hint:* Sketching an F vs. t graph might help you think about this.

Answer: The change of the momentum of the glass will be the area under the F vs. t graph. If we want the glass not to break, but the maximum Δp_y to take place, the force will have to be a flat F_{max} during the whole time interval. This means that the limit for the glass not breaking is

$$F_{\text{max}}\tau = m\left(v_{fy} - v_{iy}\right)$$

The better way to express this is that the glass will certainly break if

$$F_{\text{max}}\tau < m\left(v_{fy} - v_{iy}\right)$$

(b) Let's say the inequality you wrote down does not hold. Does this mean that the glass is certain to remain unbroken? Explain.

Answer: No, the glass can still break if the area under the curve remains smaller than $F_{\text{max}}\tau$, but at some time the force goes up above F_{max}.

2. (30 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You also work with carts that incorporate a spring that can be compressed and released during a collision, imparting the energy stored in the spring to the carts rebounding from the collision.

You set up the collision with a cart with mass $2m$ with initial velocity $v_{2i} = v$ heading toward a cart with mass m that starts at rest. You measure the final velocity of the cart with mass m in three different experiments, obtaining $v_{1f} = v$, $v_{1f} = \frac{4}{3}v$, and $v_{1f} = 2v$. Analyze these three experiments and determine which experiments must have had a compressed spring released during the collision.

Answer: Momentum conservation:

$$(2m)v + 0 = (2m)v_{2f} + mv_{1f} \quad \Rightarrow \quad v_{2f} = v - \frac{1}{2}v_{1f}$$

The change in total energy due to the collision will be the difference in initial and final total kinetic energies, since no relevant potential energies apply, and there is no loss to friction.

$$\Delta E = \frac{1}{2}(2m)v_{2f}^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Putting the result from momentum conservation in there,

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v_{1f}\right)^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Now, we need to investigate the sign of ΔE. If $\Delta E > 0$, extra energy has been added to the carts, which would be because of the spring being released.

The first experiment, with $v_{1f} = v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v\right)^2 + \frac{1}{2}mv^2 - \frac{1}{2}(2m)v^2 = -\frac{1}{4}mv^2 < 0$$

This is an inelastic collision, and since $\Delta E < 0$, there's no evidence for a spring release here.

The second experiment, with $v_{1f} = \frac{4}{3}v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}\frac{4}{3}v\right)^2 + \frac{1}{2}m\left(\frac{4}{3}v\right)^2 - \frac{1}{2}(2m)v^2 = 0$$

This could be an elastic collision, which could happen without a spring being released. So again, there's no evidence for a spring release.

The third experiment, with $v_{1f} = 2v$:

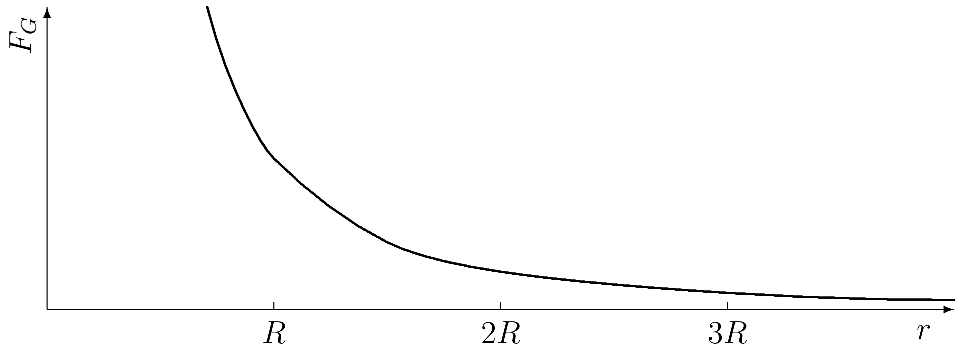
$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}2v\right)^2 + \frac{1}{2}m\,(2v)^2 - \frac{1}{2}(2m)v^2 = mv^2 > 0$$

There is no way $\Delta E > 0$ without the spring release! It must have happened in this third experiment.

3. (40 points) Remember how we got the gravitational potential energy mgh: the applied force acting against gravity had a magnitude of mg, and we found the area under the force-versus-distance curve, a rectangle of height mg and base h.

Now we want to generalize this to beyond locations close to the Earth's surface. Take the gravitational force magnitude F_G between two point masses m_1 and m_2 separated by a distance r. We will again look at the area under the force-distance curve.

(a) Sketch a graph of F_G versus r.



Now, according to your sketch, do you do more work in changing r from R to $1.1R$, from $2R$ to $2.1R$, or from $3R$ to $3.1R$?

Answer: You'll do the most work in going from R to $1.1R$, as that is the largest additional area under the curve. From $3R$ to $3.1R$ is the least work.

(b) The convention for gravitational potential energy is to say that it is zero when the masses are infinitely far from each other. So the expression for U_G must become very small as r becomes large. Given this, and the behavior you found in part (a), which of the following is the correct general equation for U_G? (Only one of the options given is consistent with what you found about U_G.)

(i) $U_G = \frac{1}{2}Gr^2$

(ii) $U_G = m_1 m_2 r$

(iii) $U_G = \frac{m_1}{m_2}e^{-Gr}$

(iv) $U_G = -\ln Gr$

(v) $U_G = -Gm_1m_2/r$. This is the only one that behaves according to (a), is very small as r becomes very large, has the proper units, and where m_1 and m_2 are interchangeable.

(c) Given your U_G, find the escape speed of an object launched away from Earth. This is the minimum speed necessary to never fall back to Earth under the influence of gravity: You start from r equal to the radius of Earth and speed equal to your escape speed, and end up at r equal to infinity and the object at rest. You can look up data about the Earth to find a numerical result.

Answer: Use energy conservation. The final energy is 0, since as $r \to \infty$, $U_G \to 0$, and the object is at rest and has no kinetic energy. The initial energy must therefore add up to zero:

$$\frac{1}{2}mv^2 - \frac{Gm_E m}{r_E} = 0$$

Solving for v, we get

$$v = \sqrt{\frac{2Gm_E}{r_E}} = 1.12 \times 10^4 \text{ m/s}$$

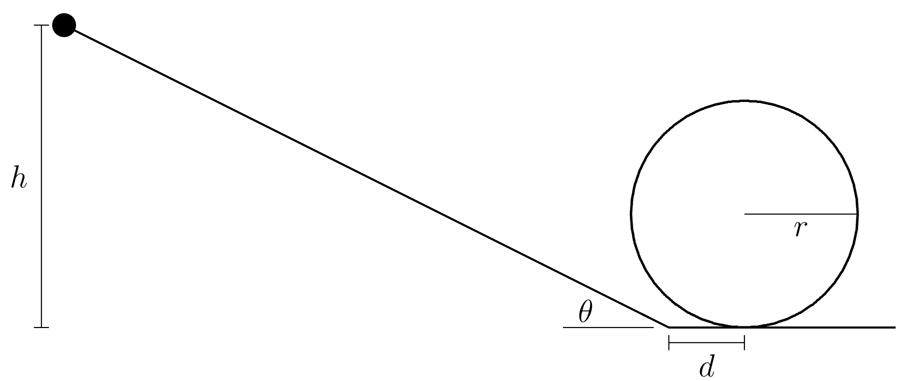
(d) Find an equation for the radius r_s for the event horizon of a black hole with mass m. The event horizon marks the point beyond which nothing can return, since it would have to travel faster than light. You find r_s by setting the escape speed equal to the speed of light c.

Answer: You just set

$$\sqrt{\frac{2Gm}{r}} = c \qquad \Rightarrow \qquad r = \frac{2Gm}{c^2}$$

Extra Problems (not graded)

4. (0 points) A roller coaster cart starts from an initial height h, goes downhill at an angle θ with the horizontal, travels a horizontal distance d, and goes through a loop with radius r.



(a) Assuming dissipative forces such as friction and drag are negligible, what is the minimum height h_{min} for the cart to be able to complete the loop?

Answer: We've previously obtained $v_{\text{min}} = \sqrt{gr}$. Using this, energy conservation gives:

$$mgh_{\text{min}} = mg(2r) + \tfrac{1}{2}m\left(\sqrt{gr}\right)^2 \quad \Rightarrow \quad h_{\text{min}} = \tfrac{5}{2}r$$

(b) Assume that the cart loses energy to dissipative forces at a rate of ε per length of track traveled: if, for example, it travels a distance l, then $E_{\text{loss}} = \varepsilon l$. In that case, what would h_{min} now be?

Answer: The total distance covered is the length of the downhill stretch, the horizontal past, and half the circumference of the circle. Therefore $E_{\text{loss}} = \varepsilon\,(h/\sin\theta + d + \pi r)$. Using this, energy conservation becomes

$$mgh_{\text{min}} = mg(2r) + \tfrac{1}{2}m\left(\sqrt{gr}\right)^2 + E_{\text{loss}} \quad \Rightarrow \quad h_{\text{min}} = \tfrac{5}{2}r + \frac{\varepsilon}{mg}\left(\frac{h_{\text{min}}}{\sin\theta} + d + \pi r\right)$$

With h_{min} on both sides, a bit more algebra gives:

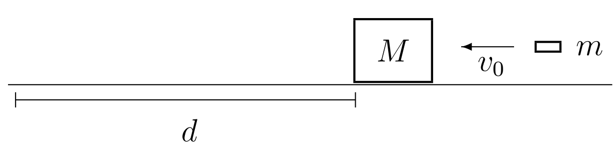
$$h_{\text{min}} = \frac{\frac{5}{2}r + \frac{\varepsilon}{mg}\,(d + \pi r)}{1 - \frac{\varepsilon}{mg\sin\theta}}$$

For $\varepsilon > 0$, $h_{\text{min}} > \frac{5}{2}r$, as it should be.

(c) Describe the ways in which the assumption of losing ε per length traveled, regardless of the orientation of the track, is not completely accurate.

Answer: With friction, $E_{\text{loss}} = -W_f = f_k l$. But the magnitude of the friction force depends on the orientation of the track: maximum when flat, and smaller as the angle increases. So at least on the circle, the loss cannot be independent of orientation. (If you work it out, the loss depends on θ on the downhill part as well, but a verbal argument is sufficient.)

5. (30 points) You decide to design a variation on the idea of a ballistic pendulum. You keep the part with a bullet with mass m and initial velocity v_0 embedding in a block of mass M. But after that totally inelastic collision, instead of having the bullet-block combination swing upwards on a pendulum, you let it slide on a rough and flat surface, with a coefficient of kinetic friction μ_k. You wait for the bullet-and-block to come to a stop, and measure the distance d it traveled from the point where the bullet first struck the block.



(a) Find an equation for v_0. *Hint:* Think about how to account for the work done by friction, which will give you the energy loss.

Answer: Use momentum conservation for the first step:

$$mv_0 = (M+m)v \qquad \Rightarrow \qquad v = \frac{m}{M+m}\,v_0$$

The kinetic energy coming out of the collision will be

$$K_i = \frac{1}{2}(M+m)\left(\frac{m}{M+m}\,v_0\right)^2 = \frac{m^2}{2(M+m)}\,v_0^2$$

Since the final kinetic energy $K_f = 0$, all this energy must go into the work done against friction. Since the kinetic friction for a flat surface is $f = \mu_k n = \mu_k(M+m)g$, we can calculate this as

$$-W_f = \mu_k(M+m)gd = E_{\text{loss}}$$

Setting these equal,

$$\mu_k(M+m)gd = \frac{m^2}{2(M+m)}\,v_0^2 \qquad \Rightarrow \qquad v_0 = \left(\frac{M+m}{m}\right)\sqrt{2\mu_k g d}$$

(b) Assume you will use your setup to measure bullet speeds of around $v_0 = 500\,\text{m/s}$, with bullet masses around $m = 0.01\,\text{kg}$. Other reasonable values would be $\mu_k = 0.5$ and $d = 1.0\,\text{m}$, which is practical to measure in the lab. What, then, would the block mass M have to be in your setup?

Answer:

$$M = m\left(\frac{v_0}{\sqrt{2\mu_k g d}} - 1\right) = 1.6\ \text{kg}$$