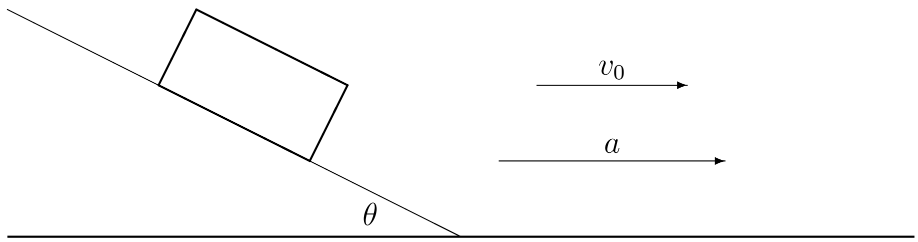
Solutions to Exam 1; Phys 185

1. (60 points) You have a block on a flat surface that is inclined by an angle θ with the horizontal. The drag force is negligible. The inclined surface itself, however, is in motion horizontally.



(a) The inclined surface is moving at a constant velocity with magnitude v_0, and there is no friction between the block and the surface. Can the block remain at rest relative to the surface if v_0 is large enough? Explain.

Answer: Constant velocity means zero acceleration, so the forces on the block need to add up to zero. But if $\theta \neq 0$, the normal force $\vec{n}$ cannot point straight up, and therefore cannot cancel out the weight $\vec{w}$. It will slide down.

(b) The surface still moves at constant velocity. But there is now friction between the block and the surface, with associated coefficients of friction μ_s and μ_k. Obtain the conditions for the block remaining at rest relative to the surface.

Answer: Since $\vec{a} = 0$, the situation is the same as a stationary block. Draw the usual tilted coordinate axes, with $+x$ pointing down-slope. The normal force takes care of $a_y = 0$, and therefore $n = mg\cos\theta$. Parallel to the surface, $mg\sin\theta - f_s = ma_x$. For $a_x = 0$, we need the friction. The maximum possible friction force is $\mu_s mg\cos\theta$, so at the limit, $mg\sin\theta = \mu_s mg\cos\theta$. Therefore, if

$$\mu_s \geq \frac{\sin\theta}{\cos\theta} = \tan\theta$$

the block will not slide down.

(c) The inclined surface now moves horizontally at a constant acceleration $a > 0$, and there is no friction between the block and the surface. Again, describe the conditions for the block remaining at rest relative to the surface.

Answer: Since the acceleration is horizontal, it will be most convenient *not* to tilt the axes. In that case, in the vertical direction, $n\cos\theta - mg = ma_y = 0$. Horizontally,

$n \sin\theta = ma_x = ma$. Putting these together,

$$\frac{mg}{\cos\theta}\sin\theta = ma \quad \Rightarrow \quad g\tan\theta = a$$

The block will be at relative rest only at a particular angle for each acceleration a.

(d) The surface still moves at constant acceleration. But there is now friction between the block and the surface, with associated coefficients of friction μ_s and μ_k. Obtain the conditions for the block not sliding *down* relative to the surface.

Answer: If friction is to prevent sliding down, it must be acting up-slope. Look at the maximum possible $f_s = \mu_s n$:

$$\sum F_y = n\cos\theta + \mu_s n \sin\theta - mg = 0 \quad \Rightarrow \quad n = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

$$\sum F_x = n\sin\theta - \mu_s n\cos\theta = ma \quad \Rightarrow \quad g\frac{sin\theta - \mu_s \sin\theta}{\cos\theta + \mu_s\sin\theta} = a$$

If the acceleration is *smaller*, the block will slide, so

$$a \geq g\left(\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}\right)$$

is the condition for not sliding down. For small θ, notice that the result will be negative, so that any $a > 0$ will suffice.

(e) The surface still moves at constant acceleration. But there is now friction between the block and the surface, with associated coefficients of friction μ_s and μ_k. Obtain the conditions for the block not sliding *up* relative to the surface.

Answer: If friction is to prevent sliding up, it must be acting down-slope. Otherwise, the calculation is identical to what we di for sliding down. So all we have to do is to flip the friction, which can be accomplished by changing μ_s to $-\mu_s$ everywhere in the result, and notice that this is now an upper (rather than lower) limit for the acceleration:

$$a \leq g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

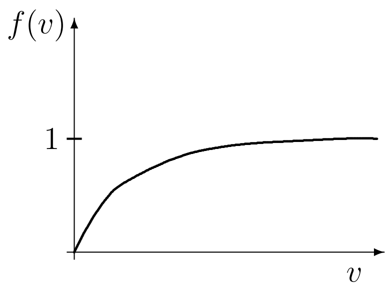
is the condition for not sliding up.

(f) Let's say θ starts from 0, and you then increase it in successive experiments. Will you ever reach an angle where the block cannot slide up, no matter how large a is?

Answer: As θ increases, $\cos\theta$ goes down from 1, and $\sin\theta$ increases from 0. Therefore, the upper limit for a is obtained by dividing by a smaller and smaller number, until, when

$\cos\theta - \mu_s \sin\theta = 0$, you divide by zero, reaching infinity. At that point and beyond, a can be as large as you want, and you'll never have the block slide up. So for angles $\theta \geq \cot^{-1}\mu_s$, the block can't slide up.

2. (40 points) While exploring space, you land on a planet with a peculiar atmosphere, where the drag force behaves differently from Earth. After some experimentation, you establish that $\vec{D}$ has a direction opposing the velocity, and the magnitude $D = \kappa f(v)$. Here, κ has a value that depends on the shape and size of the object and what its surface is made of, while $f(v)$ is a rising function of speed that approaches the value 1 at moderate speeds. Also, the acceleration due to gravity on this planet has a value of $g = 8.0\,\mathrm{m/s^2}$.



(a) What is an appropriate unit for κ?

Answer: Since $f(v)$ is unitless (1 has no units), and D is a force, κ must have force units: N.

(b) In this planet's atmosphere, you drop an object from rest at an enormous height. The object has $m = 2.0\,\mathrm{kg}$ and $\kappa = 21.0$ (in the appropriate standard SI units). Sketch qualitative graphs for a_y vs t and v_y vs t. If you can calculate any numerical values for a_y or v_y after the object has been falling for a while, do so.

Answer: As the object falls, its speed v will increase, and so will $f(v)$. The total force on the object is

$$\sum F_y = D - w = \kappa f(v) - mg = ma_y$$

If the object approaches a speed where D is almost w, then the force and therefore the acceleration of the object will approach zero. This is terminal velocity. In this case, $w = mg = 16.0\,\mathrm{N}$. D starts at 0, but can go anywhere up to $(21\,\mathrm{N})1 = 21.0\,\mathrm{N}$. Therefore, it is possible that $D - w = 0$, and a terminal velocity will exist. You can't calculate that from the graph.

Just draw a v_y–t graph with v_y starting at 0 and dropping down towards a constant but unspecified $-v_T$ value. The a_y graph will start at $-g$ and rise toward 0.

(c) In this planet's atmosphere, you drop an object from rest at an enormous height. The object has $m = 2.0\,\mathrm{kg}$ and $\kappa = 2.10$ (in the appropriate standard SI units). Sketch qualitative graphs for a_y vs t and v_y vs t. If you can calculate any numerical values for a_y or v_y after the object has been falling for a while, do so.

Answer: With such a low κ, D starts at 0, but can only go up to $(2.10\,\mathrm{N})1 = 2.10\,\mathrm{N}$. So it can never cancel out the weight. There is no terminal velocity, but instead, a terminal acceleration

$$\kappa f(v) - mg = ma_y \quad \Rightarrow \quad a_y = \frac{\kappa}{m} - g = -6.95\,\mathrm{m/s}^2$$

The a_y graph will start at $-g$ and rise toward this terminal acceleration. The v_yb graph will not level off, but approach a straight line with the terminal acceleration as its slope.

(d) What is the condition for an object approaching a terminal velocity after a long time?

Answer: It all depends on whether D can ever equal w. The maximum value for D is κ. Therefore the condition for having a terminal velocity is

$$\kappa \geq mg$$