Solutions 7; Phys 191 Name ________________

1. (60 points) You have a single-loop circuit with an AC voltage source $V_s = V_0\, e^{i\omega t}$ together with a capacitor C, a resistor R, and an inductor L. The voltage across an inductor is $V_L = L\, dI/dt$. Set up the differential equation for Q, the charge on the capacitor. Solve for Q_p, the non-transient behavior, and also find I_p, the corresponding current.

Answer: With $I = dQ/dt$, $dI/dt = d^2Q/dt^2$. So

$$V_0\, e^{i\omega t} = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C}$$

Seek exponential solutions: $Q_p = A\, e^{i\omega t}$:

$$V_0\, e^{i\omega t} = -LA\omega^2\, e^{i\omega t} + iRA\omega\, e^{i\omega t} + \frac{A\, e^{i\omega t}}{C} \qquad \Rightarrow \qquad A = \frac{CV_0}{1 - LC\omega^2 + iRC\omega}$$

The current is

$$I_p = \frac{d}{dt}Q_p = \frac{i\omega CV_0}{1 - LC\omega^2 + iRC\omega}\, e^{i\omega t}$$

2. (40 points) What is the resonance frequency for this circuit, where I_p is a maximum?

Answer: Find the maximum by setting $\partial|I_p|/\partial\omega = 0$. Setting $\partial|I_p|^2/\partial\omega = 0$ will also work, and is slightly easier:

$$\frac{\partial|I_p|^2}{\omega} = \frac{\partial}{\partial\omega}\left[\frac{\omega^2 C^2 V_0}{\left(1 - LC\omega^2\right)^2 - R^2C^2\omega^2}\right] = 0 \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$

Note that the resonance frequency does not depend on R.