Solutions 8; Phys 191 Name ______________

1. (40 points) Use the Biot–Savart law to calculate the magnetic field of a long (infinite) straight current-carrying wire. Set up and perform the integral. You should get the known answer $\vec{B} = (\mu_0 I/2\pi r)\hat{\boldsymbol{\phi}}$, where $\hat{\boldsymbol{\phi}} = (x\,\hat{\mathbf{y}} - y\,\hat{\mathbf{x}})/r$ and $r = (x^2 + y^2)^{1/2}$.

Answer: Say the wire is along the z axis. In that case, $d\vec{l} = dz\,\hat{\mathbf{z}}$. Say we want to find $\vec{B}$ at a point on the $z = 0$ plane; after all, it doesn't matter because of symmetry. Then $\vec{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} - z\,\hat{\mathbf{z}}$. Using these,

$$d\vec{l} \times \hat{\mathbf{r}} = dz\,\hat{\mathbf{z}} \times \frac{1}{\sqrt{x^2 + y^2 + z^2}}\,(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} - z\,\hat{\mathbf{z}}) = dz\frac{x\,\hat{\mathbf{y}} - y\,\hat{\mathbf{x}}}{\sqrt{x^2 + y^2 + z^2}}$$

Biot-Savart:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} dz\frac{x\,\hat{\mathbf{y}} - y\,\hat{\mathbf{x}}}{(x^2 + y^2 + z^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{2(x\,\hat{\mathbf{y}} - y\,\hat{\mathbf{x}})}{(x^2 + y^2)} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\phi}}$$

2. (60 points) Use Ampère's Law to calculate the magnetic field both inside and outside a long, tightly-wound coil of wire (a solenoid) with n turns per unit length carrying current I. Hints: the B field must point along the axis of symmetry (explain why); use a rectangular loop as your integration path to pick up the B field values at two points; stretch one side of the loop very far away where the B field must be zero (or nearly).

Answer: Let the axis of symmetry be z. If $B_x \neq 0$ or $B_y \neq 0$, if you looked at the mirror image of the solenoid (reflection symmetry), the physical setup would be the same but $\vec{B}$ would be different. Therefore, only $B_z \neq 0$.

Using the suggested loop, with length l along the z-axis *inside* the solenoid,

$$\oint_{\text{loop}} d\vec{l} \cdot \vec{B} = lB_z = \mu_0 nlI \quad \Rightarrow \quad B_z = \mu_0 nI$$

With the whole loop *outside*, we get

$$lB_z = 0 \quad \Rightarrow \quad B_z = 0$$

for the field outside the solenoid.